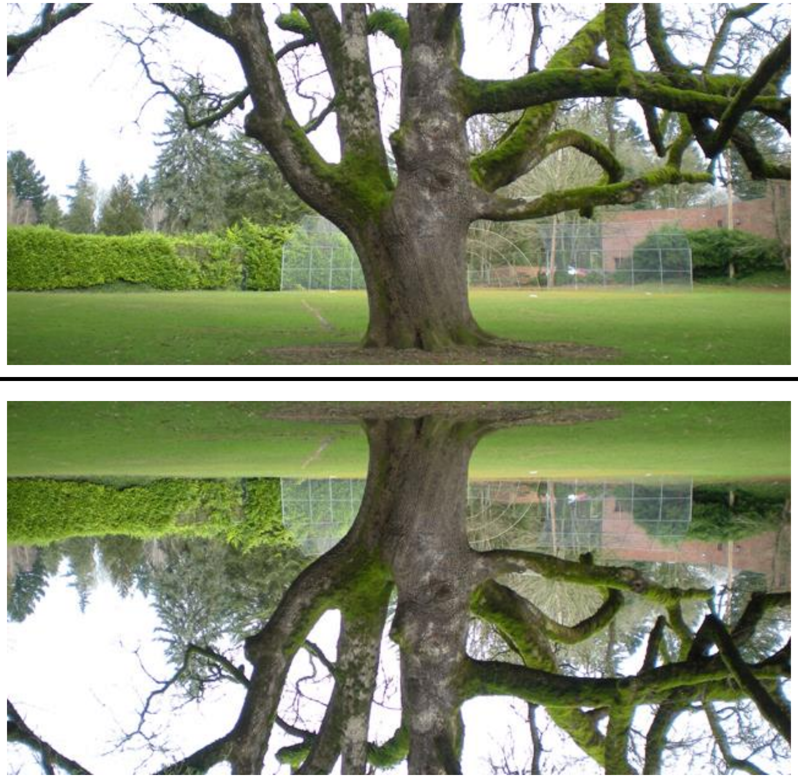
Trees

Chapter 4



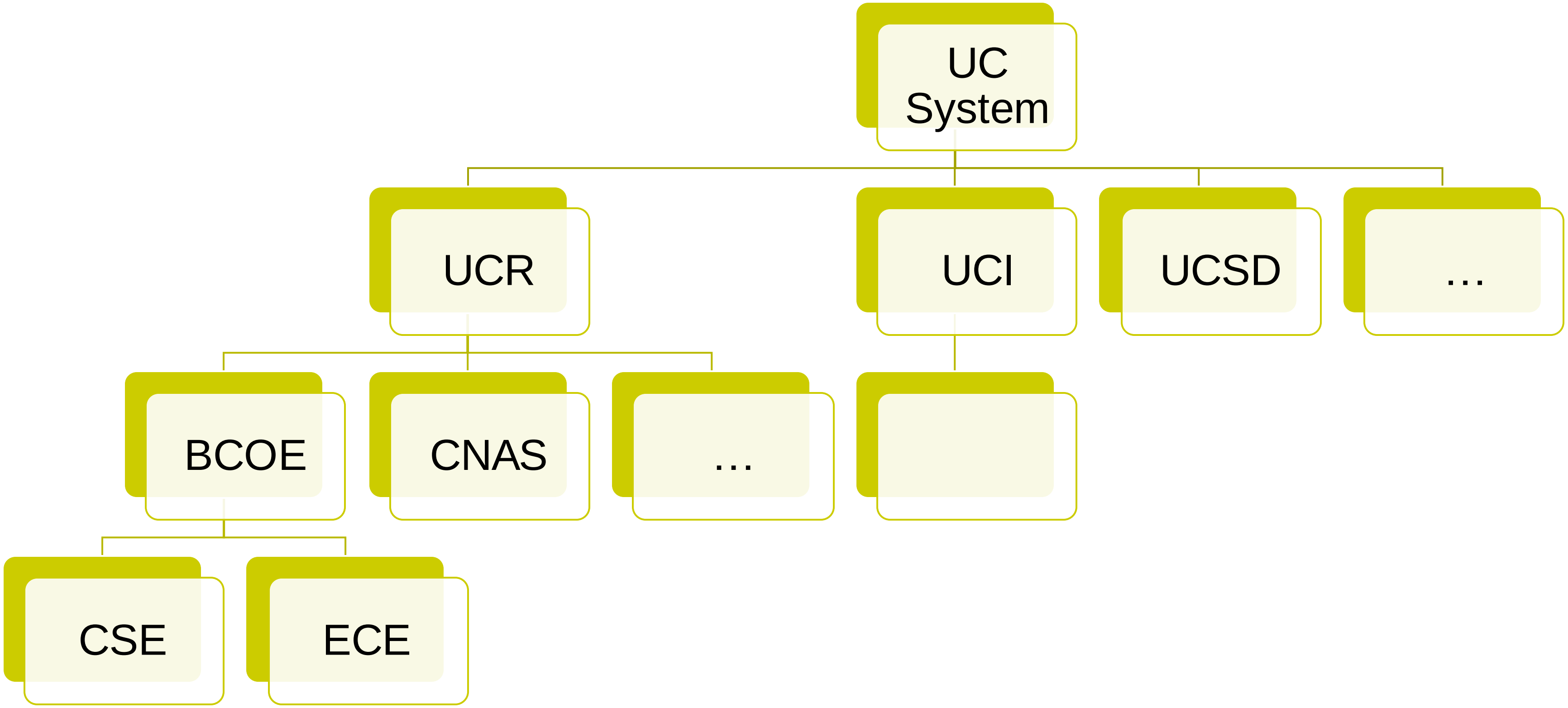
Objectives

- Understand the terminology of the tree data structure
- Represent a tree structure in a program
- Understand the importance of the binary trees
- Use a binary search tree for storing ordered elements

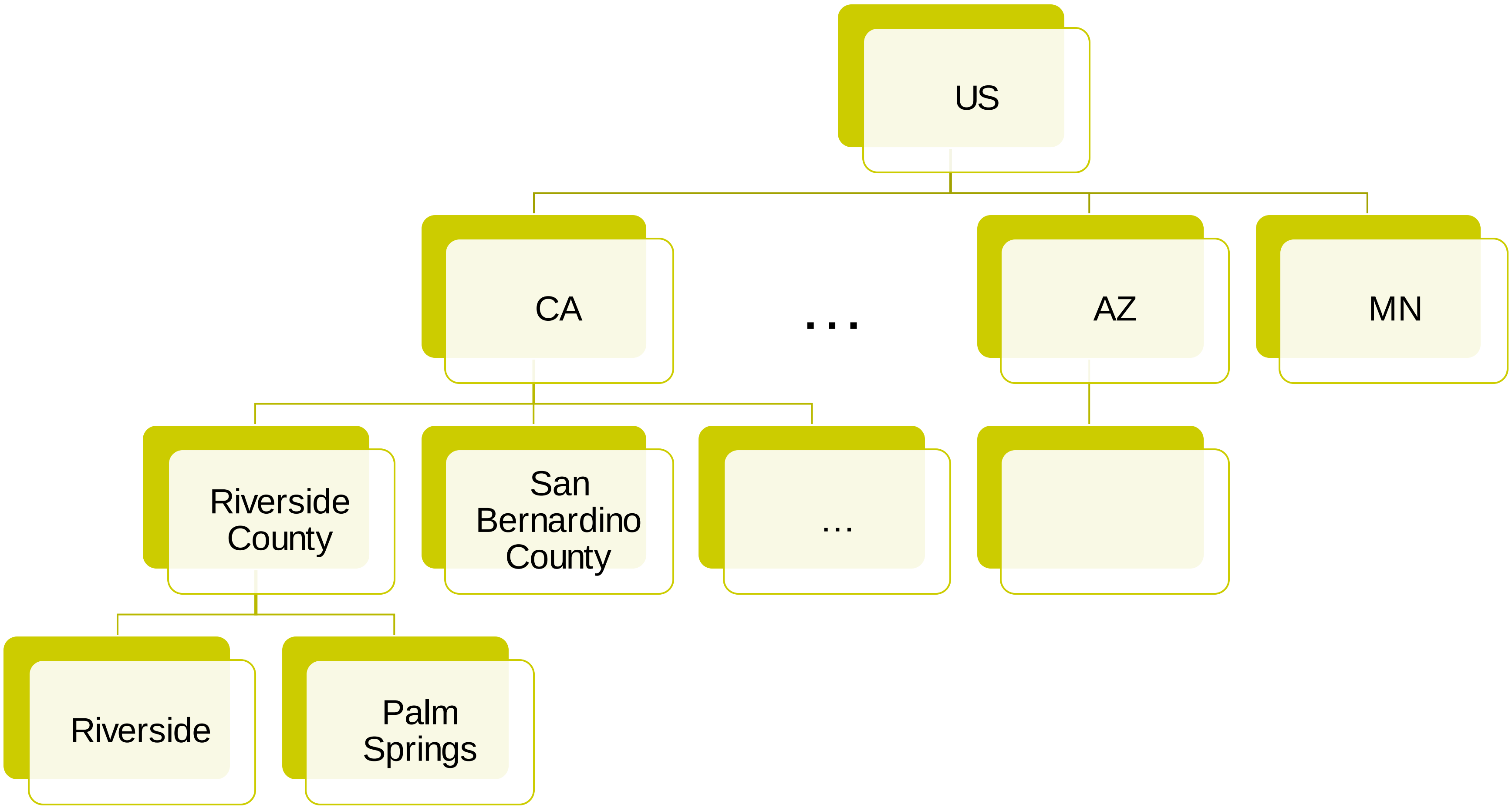
Motivation

- Why lists, stacks, and queues are not enough?
- Not everything can be linearized. We may need to represent hierarchies, for example.
- Sorted array search: O(log(n))
- Sorted array insert: O(n)
- Linked list search: O(n)
- Linked list insert: O(1)
- Can we build a data structure that is fast for both search and insert?

Hierarchical Structures



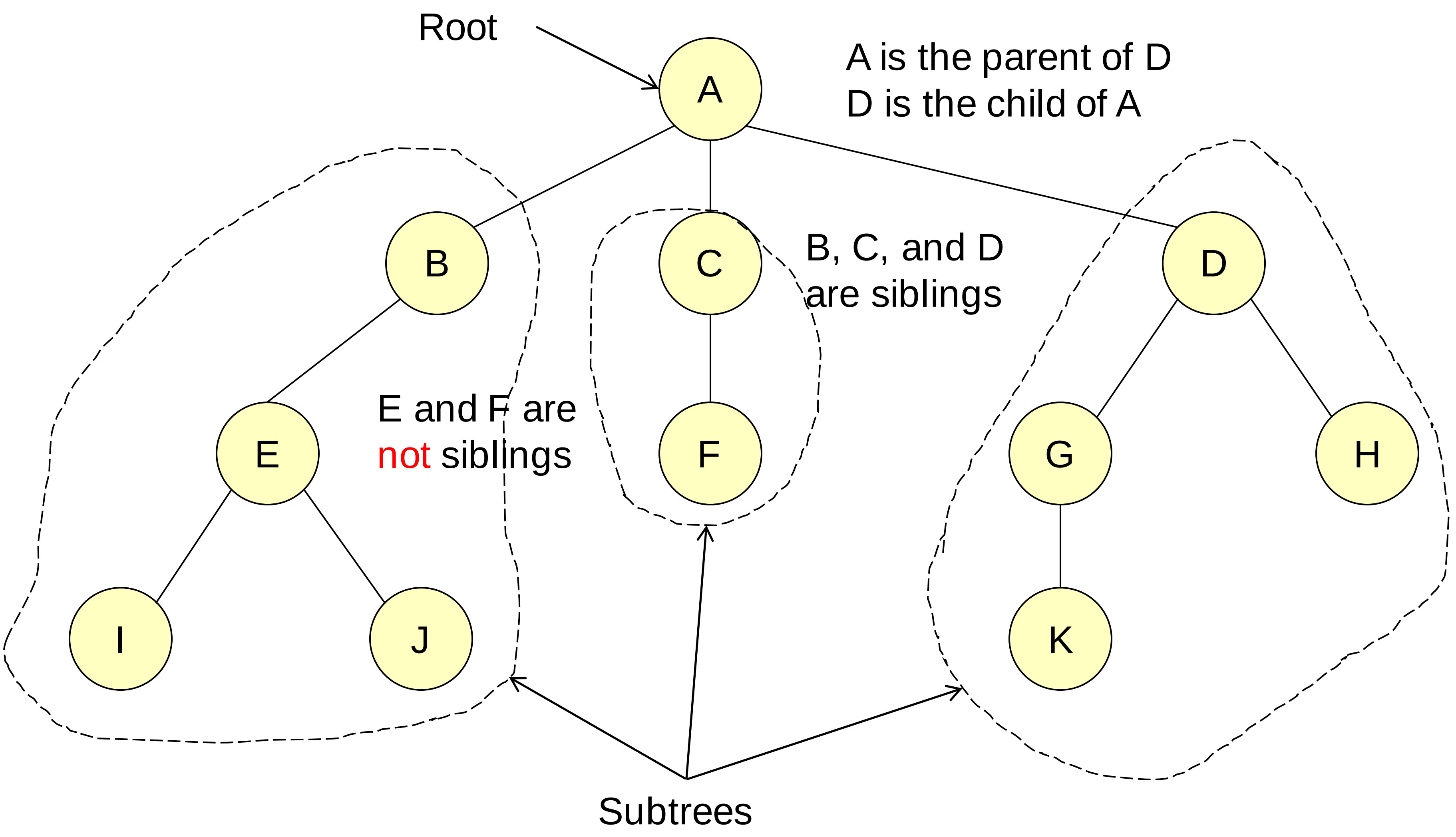
Hierarchical Structures



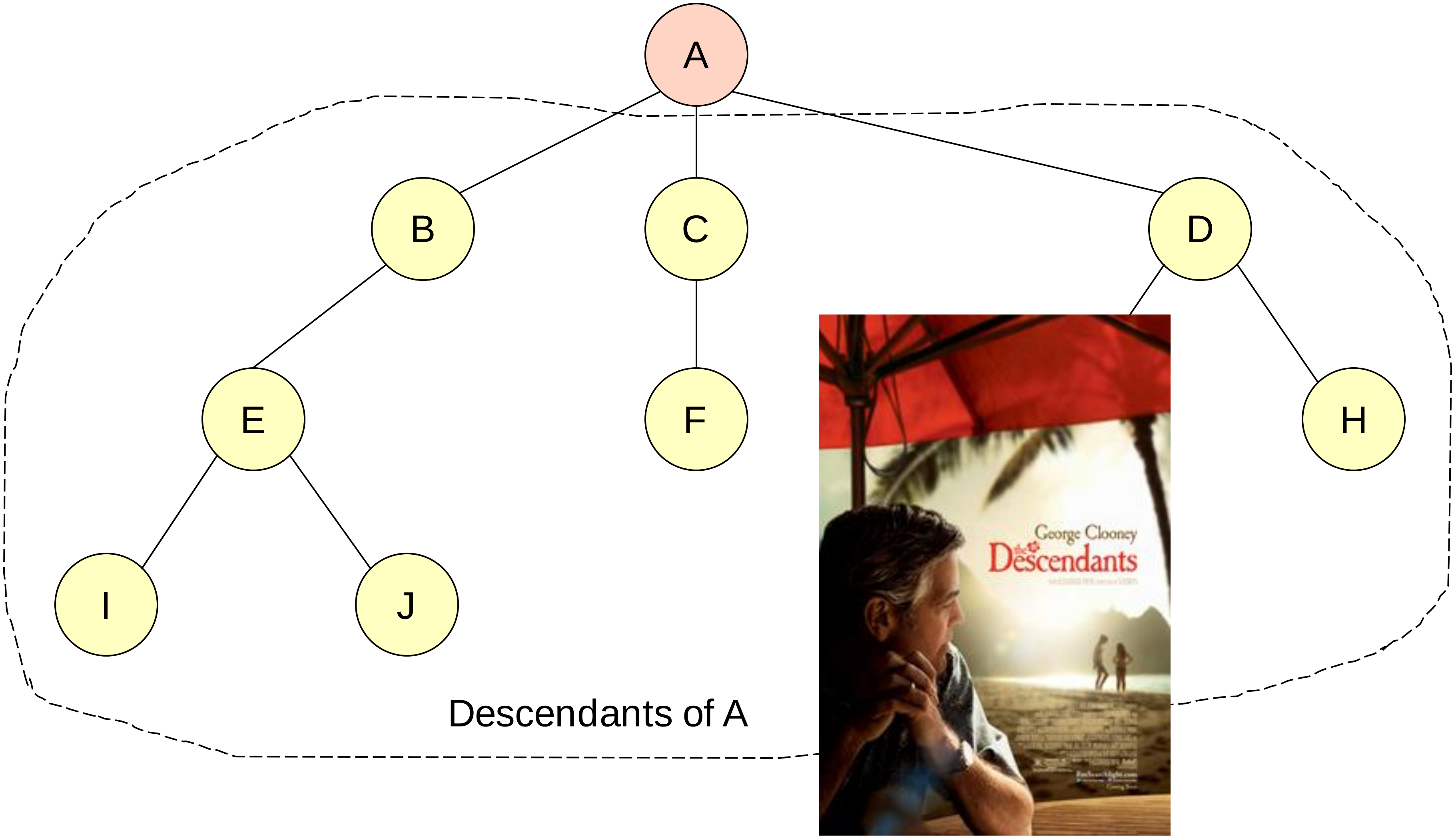
Definition

- A tree can be defined recursively
- A tree is a group of nodes
- Each node contains a value
- If the tree is not empty, one node is identified as the **root node**
- The root node has zero or more **subtrees**
- The root of a subtree is connected to the root of the tree

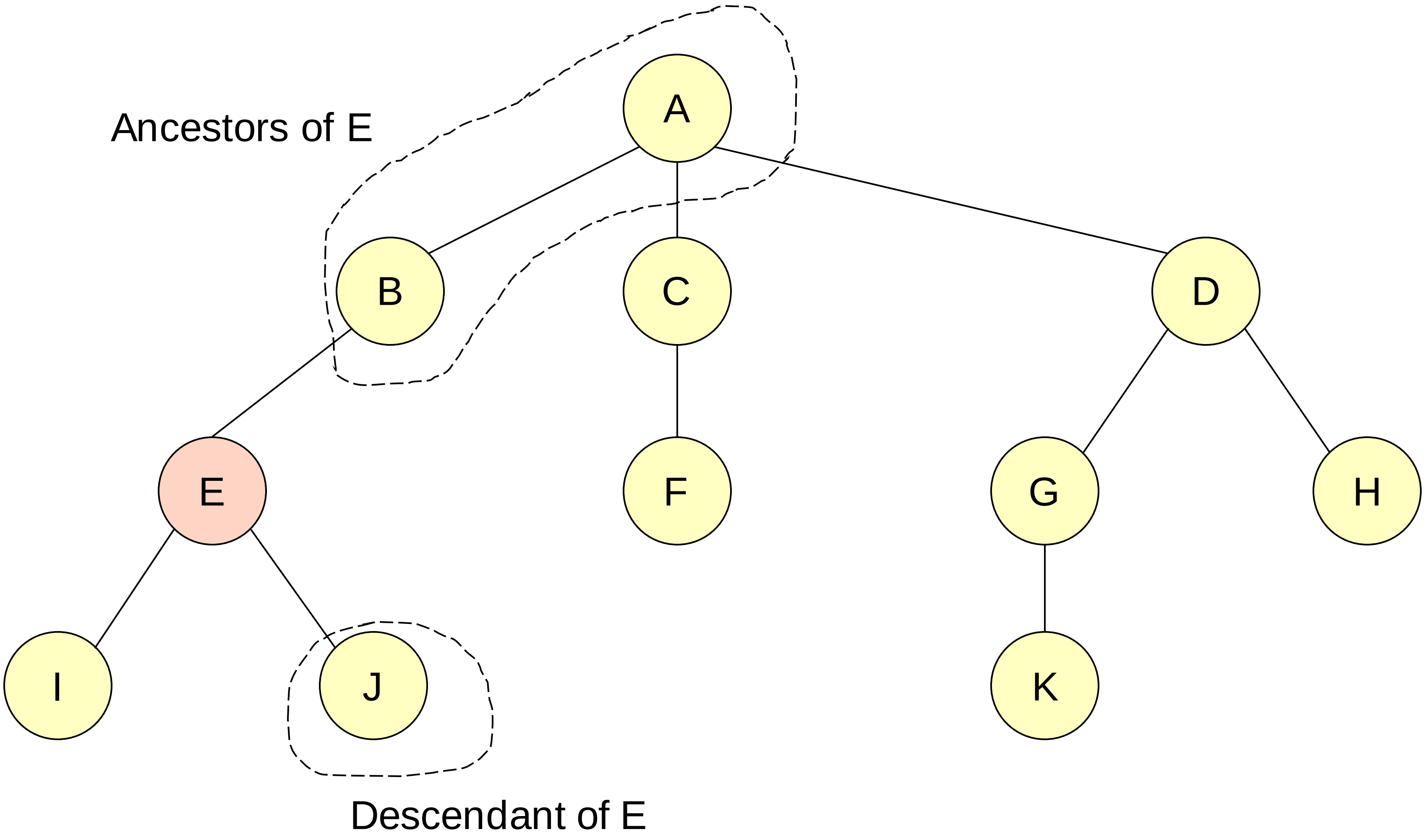
Terminology: Basic Definitions



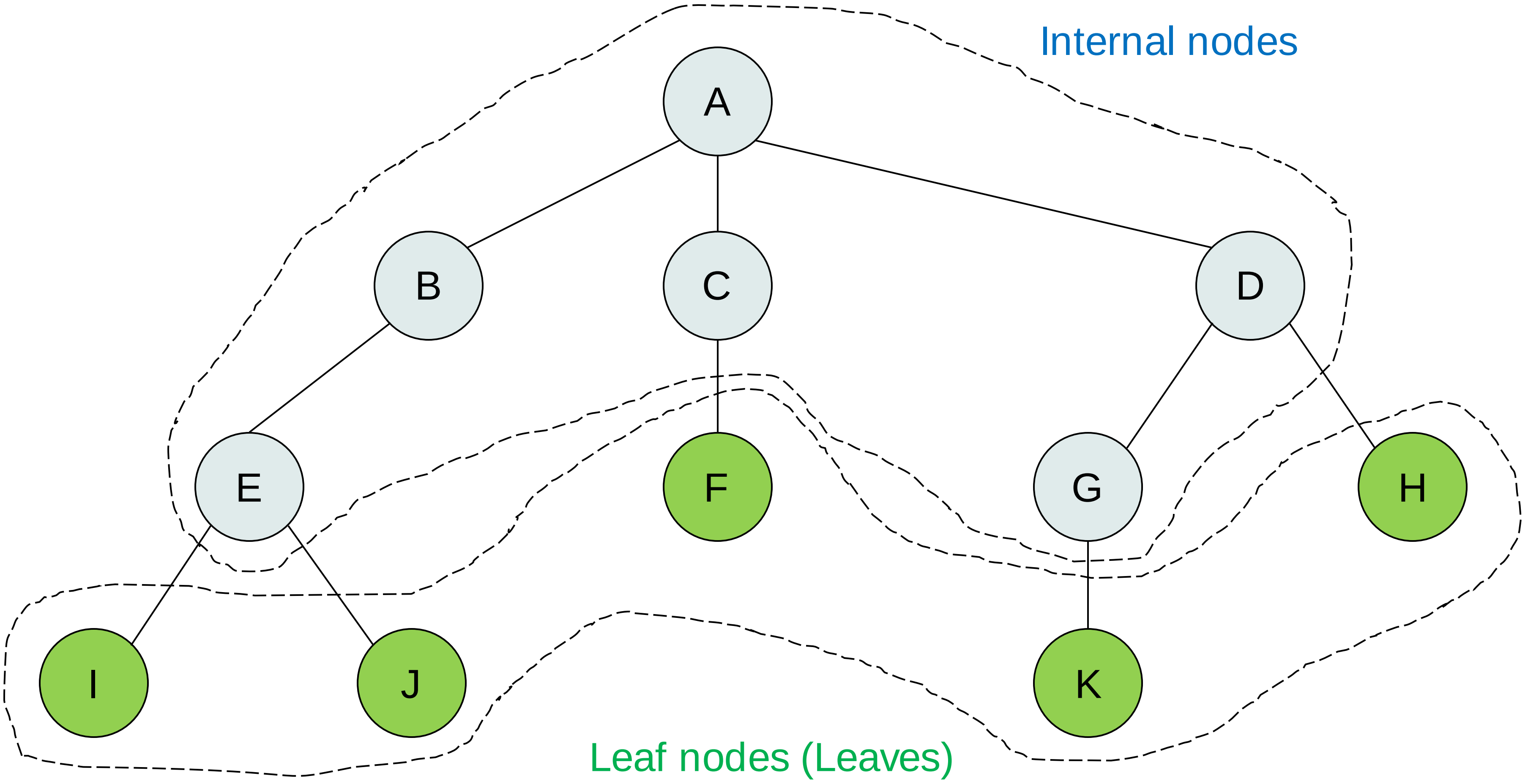
Terminology: Descendants



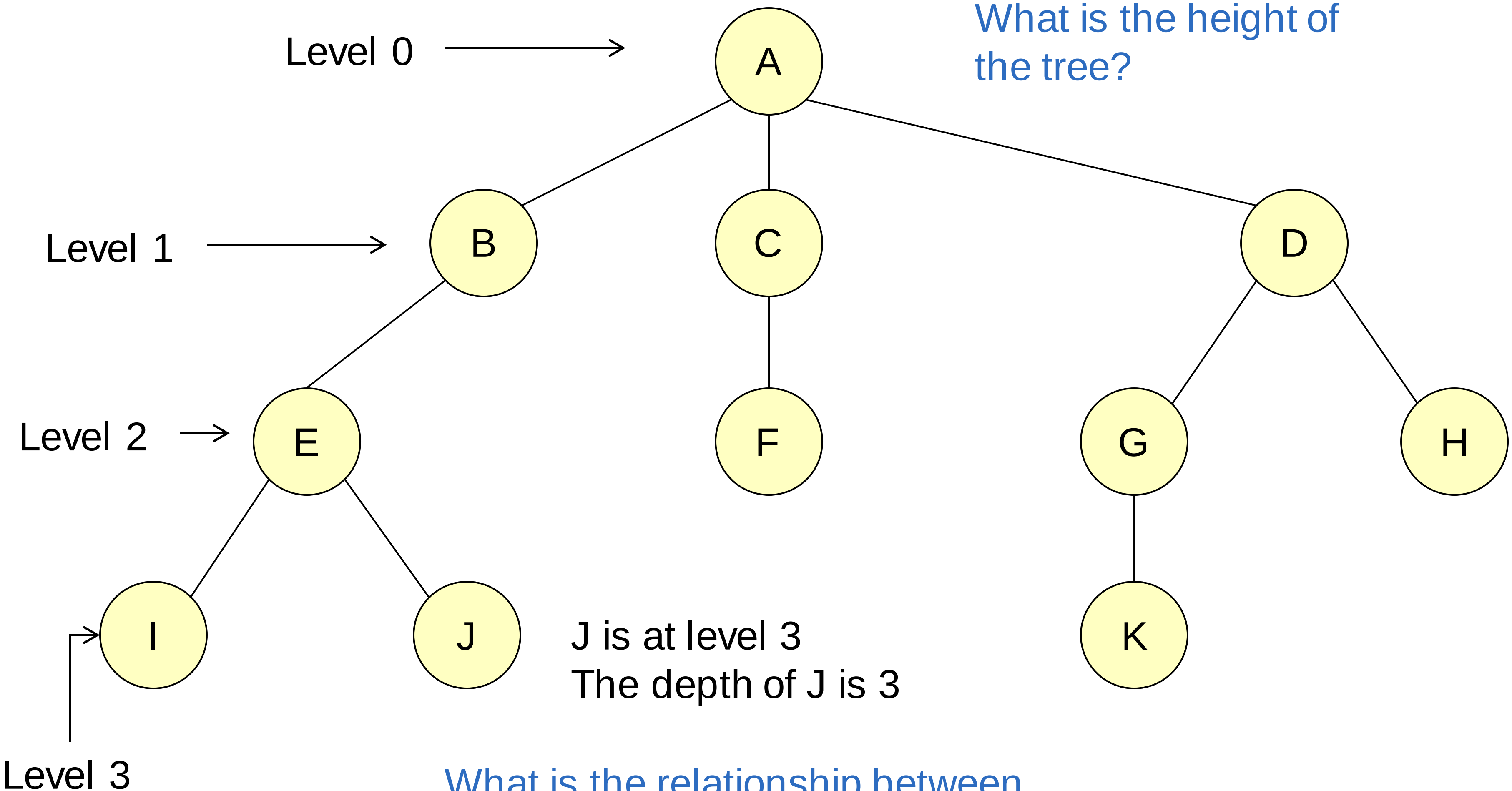
Terminology: Ancestors



Terminology: Leaves



Terminology: Levels, Depth



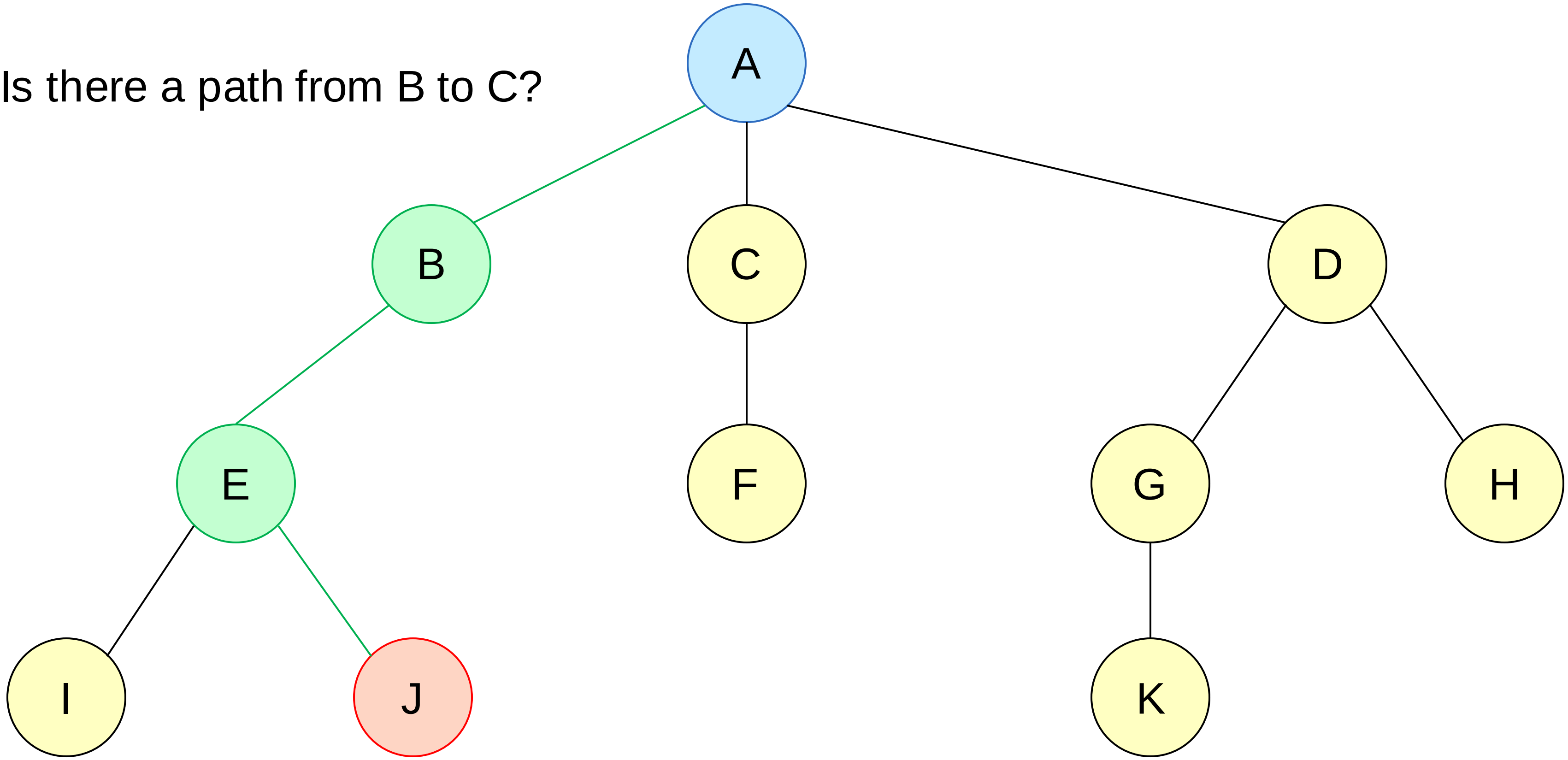
What is the height of the tree?

J is at level 3
The depth of J is 3

What is the relationship between the depth of a node and the number of ancestors?

Terminology: Path

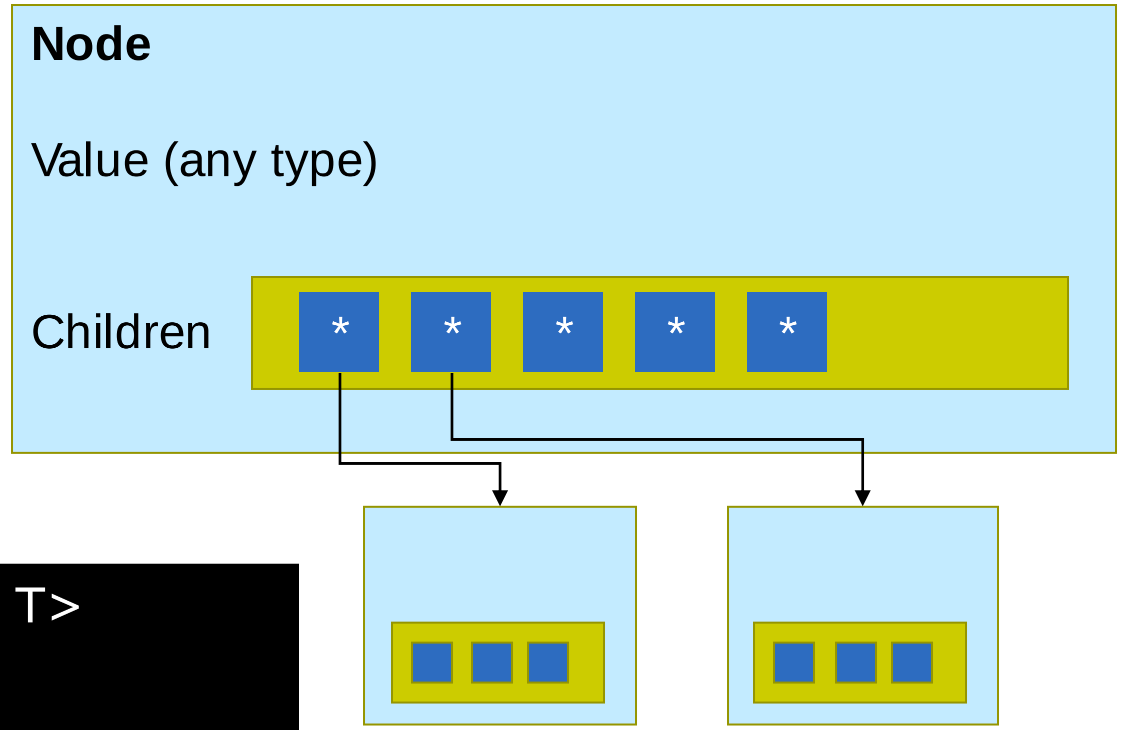
Is there a path from B to C?



The path from A to J is (A, B, E, J)
The length of the path is three (edges)

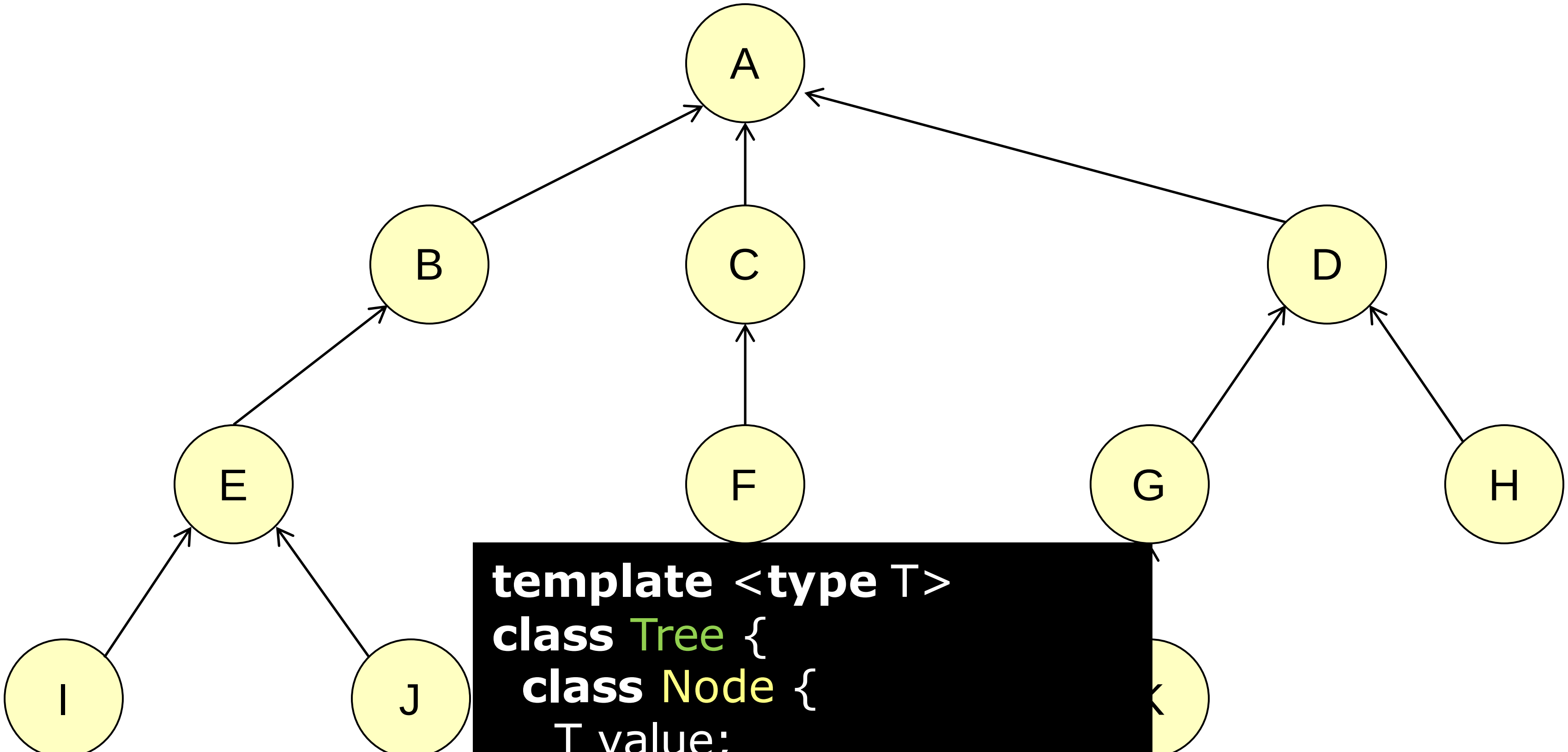
What is the path from D to K?

Tree Representation



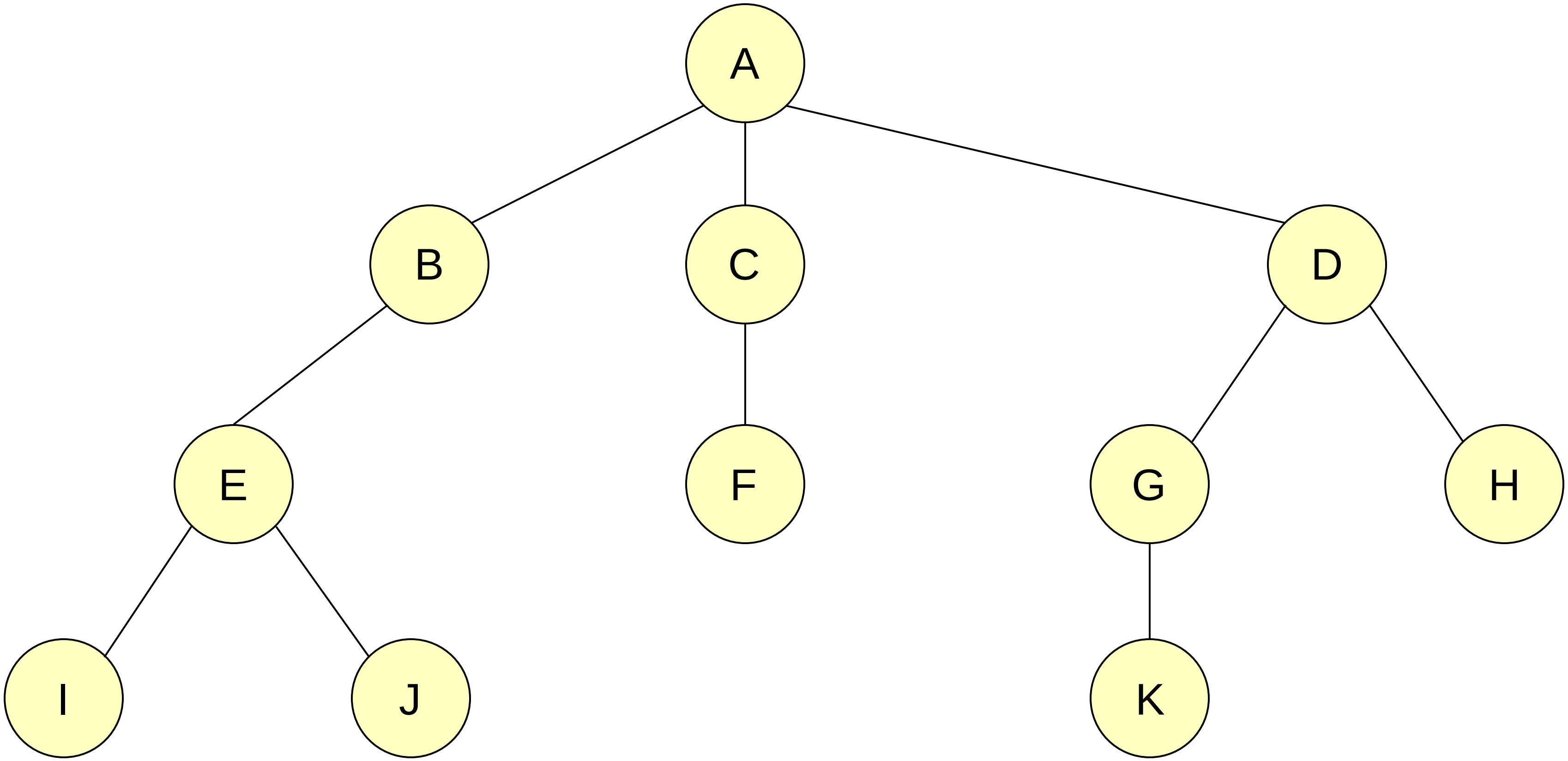

```
template <type T>
class Tree {
  class Node {
    T value;
    list<Node*> children;
  };
  Node* root;
};
```

Parent Representation

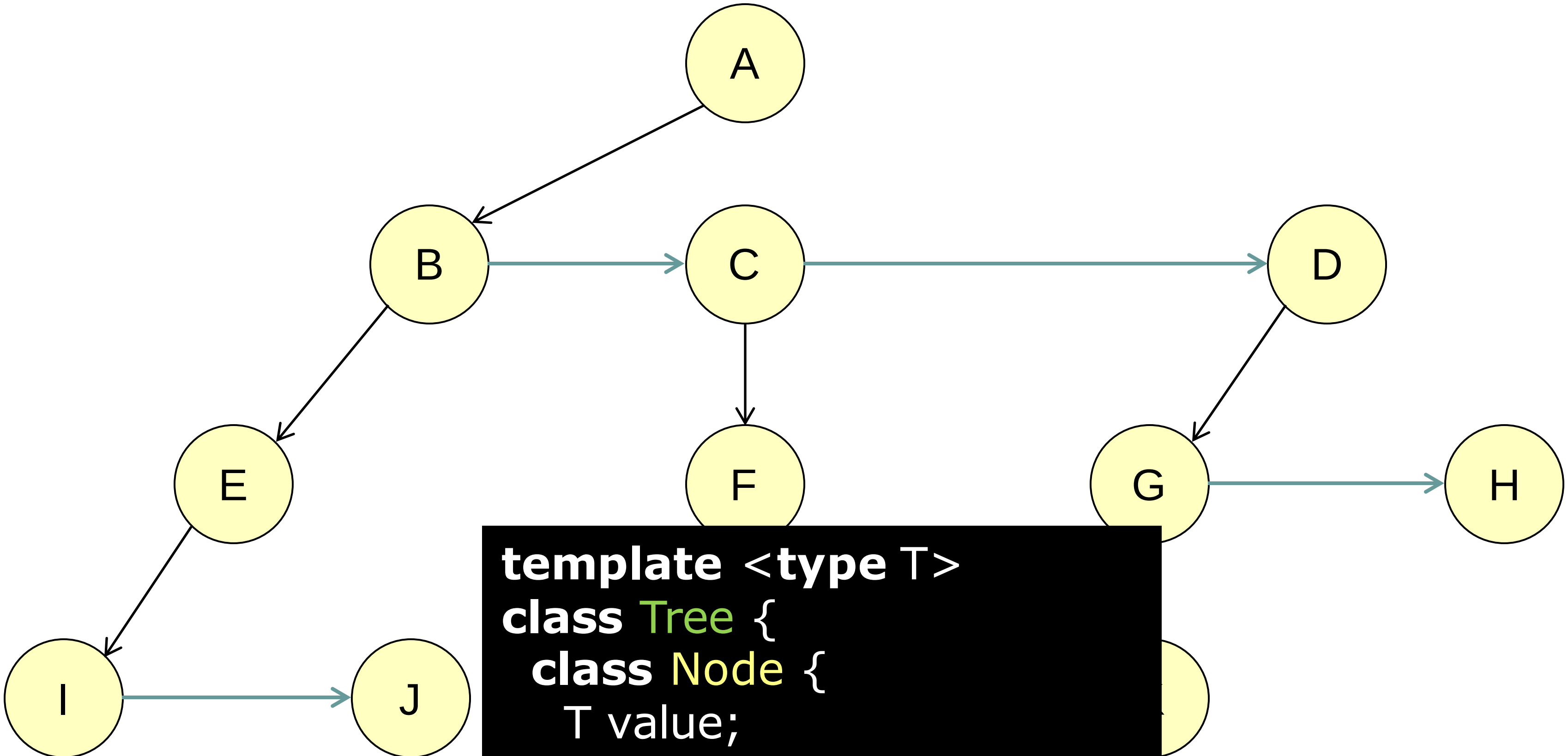



```
template <type T>
class Tree {
  class Node {
    T value;
    Node* parent;
  };
  list<Node*> nodes;
};
```

Left-child Right-sibling



Left-child Right-sibling




```
template <type T>
class Tree {
  class Node {
    T value;
    Node* left_child;
    Node* right_sibling;
  };
  Node* root;
};
```

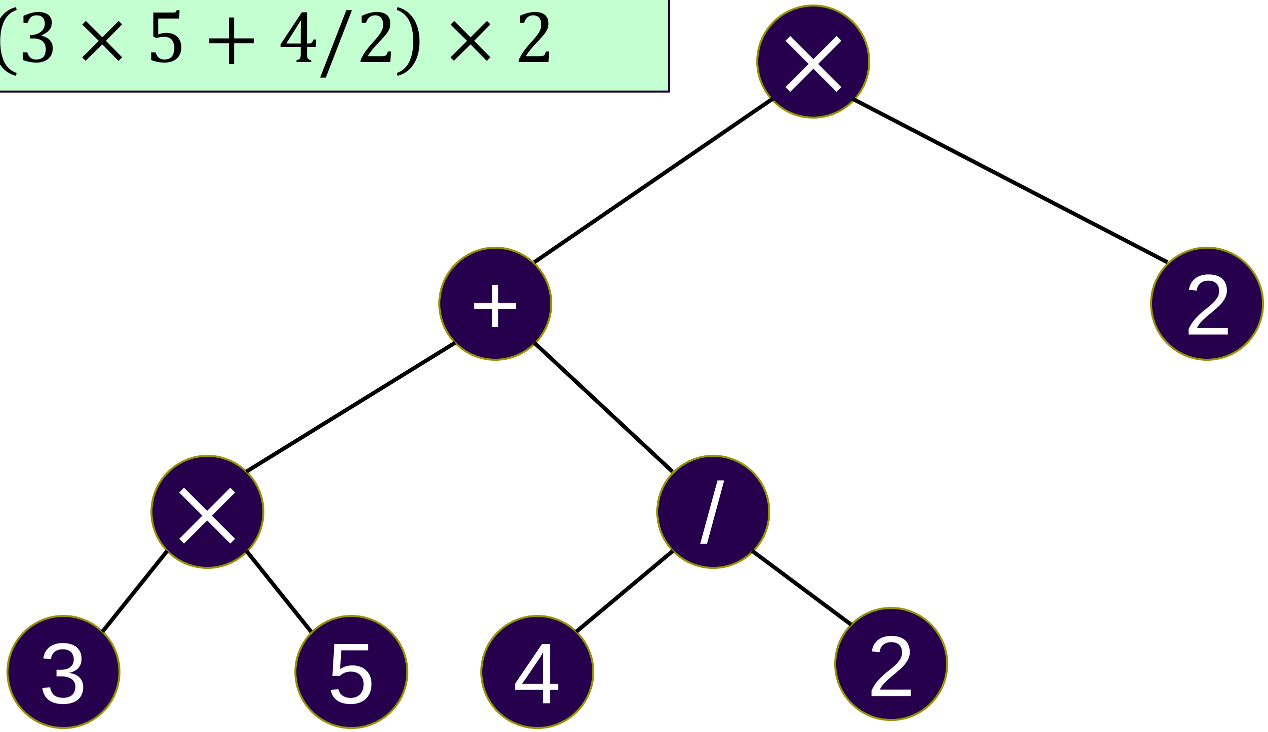
Binary Trees

- A special case where every node has at most two children
- Has many applications that make it particularly interesting
- More restricted → Room for optimization

```
template <type T>
class Tree {
  class Node {
    T value;
    Node* left;
    Node* right;
  };
  Node* root;
};
```

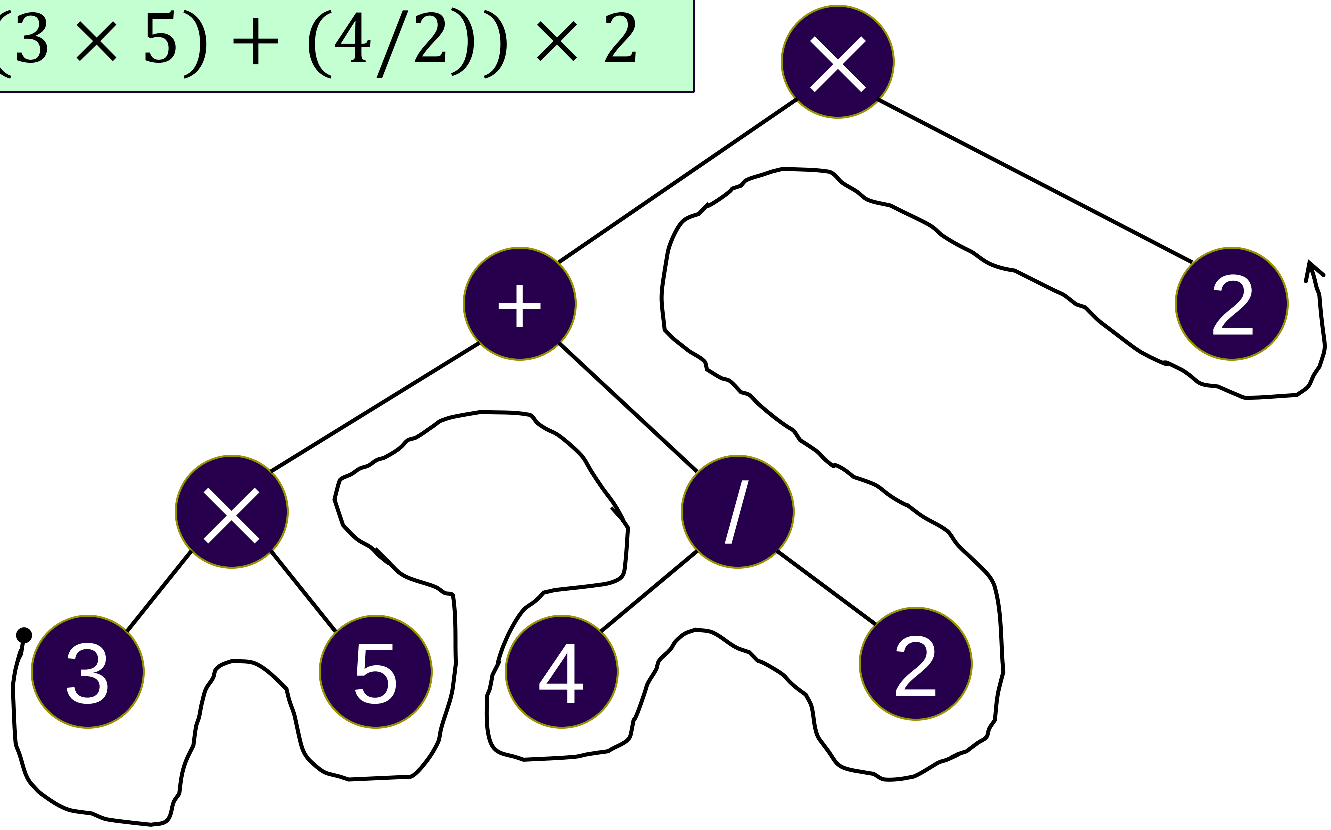
Application: Expression Tree

$$(3 \times 5 + 4/2) \times 2$$



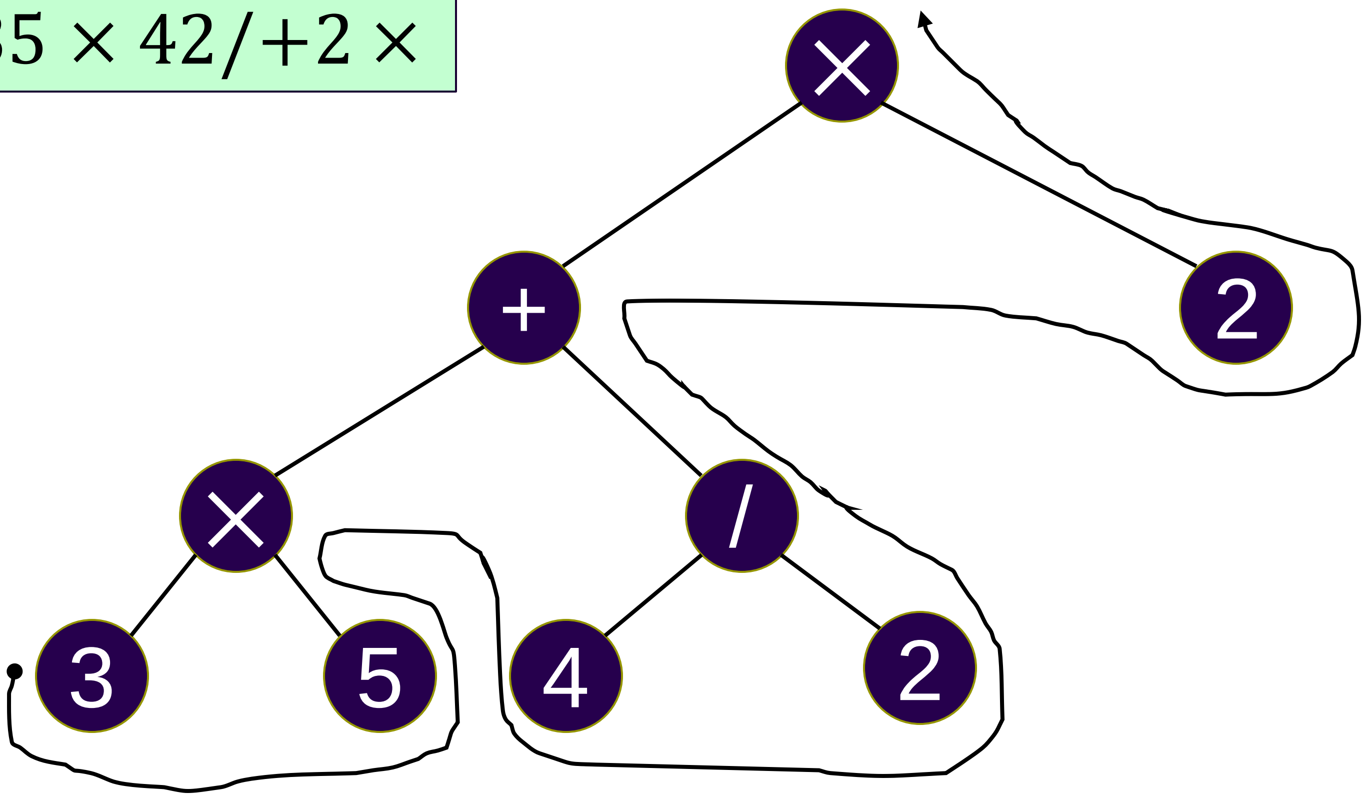
Inorder Tree Traversal

$$((3 \times 5) + (4/2)) \times 2$$



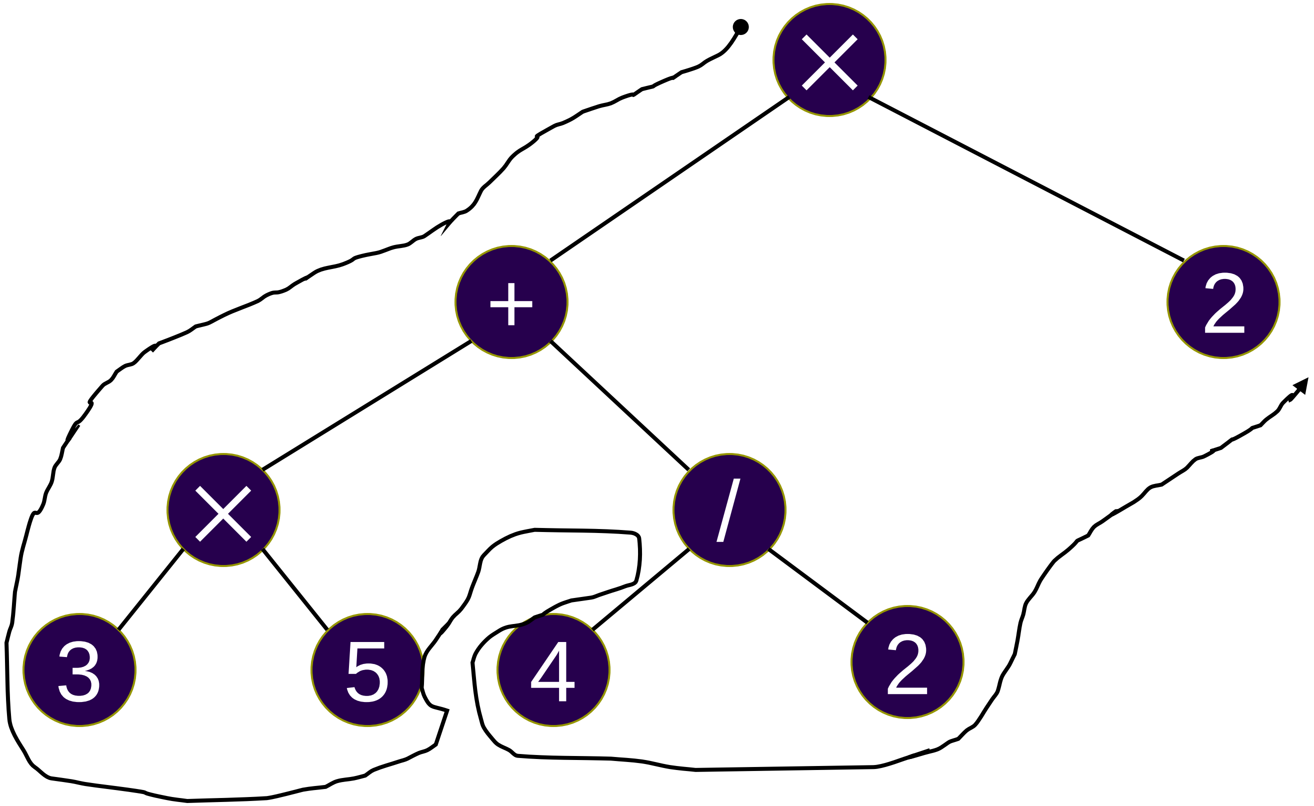
Postorder Tree Traversal

$35 \times 42/+2 \times$



Preorder Tree Traversal

×+×35/422



Implementation of Traversals

```
inorder(Node* root) {
  if (root == null)
     return;
  inorder(root->left);
  print(root->value);
  inorder(root->right);
}
```

```
postorder(Node* root) {
  if (root == null)
     return;
  postorder(root->left);
  postorder(root->right);
  print(root->value);
}
```

```
preorder(Node* root) {
  if (root == null)
     return;
  print(root->value);
  preorder(root->left);
  preorder(root->right);
}
```