CS120A – Homework #1

Spring 2003. Professor Hwang

Given April 10, 2003. Due April 17, 2003 at the beginning of class.

No late homework accepted.

Your work must be completely typeset with a word processor. Circuit diagrams can be drawn using any drawing program or by hand but it must be very neat. Handwritten works will **NOT** be accepted. (15 points total)

1. We said that XOR is the inverse of XNOR, but this is not always true for some number of inputs n. For instance, XOR is equal to XNOR for $n = 3$. Show using Boolean algebra that XOR = XNOR for $n = 3$. In general, what are the values of n where XOR = XNOR? (3)

Answer

$$
\begin{aligned}
x \oplus y \oplus z &= (x \oplus y) \oplus z \\
&= (x'y + xy') \oplus z \\
&= (x'y + xy')'\, z + (x'y + xy')\, z' \\
&= (x'y)' \cdot (xy')'\, z + x'yz' + xy'z' \\
&= (x + y') \cdot (x' + y)\, z + x'yz' + xy'z' \\
&= \cancel{xx'z} + xyz + x'y'z + \cancel{y'yz} + x'yz' + xy'z' \\
&= (xy + x'y')\, z + (x'y + xy')\, z' \\
&= (xy + x'y')\, z + (xy + x'y')'\, z' \\
&= (x \odot y)\, z + (x \odot y)'\, z' \\
&= x \odot y \odot z
\end{aligned}
$$

XOR = XNOR when n is an odd number

2. Given the following truth table with four inputs (s_2, s_1, s_0, B), derive the Boolean equation for the output function F. (3)

s_2	s_1	s_0	F
0	0	0	1
0	0	1	1
0	1	0	B'
0	1	1	B
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	0

Answer

$F = s_2's_1's_0' + s_2's_1's_0 + B's_2's_1s_0' + Bs_2's_1s_0 + s_2s_1's_0$

3. Derive the truth table for the function

$$F = ((A+B') \bullet (BC)') \odot (A \oplus B') \quad (3)$$

Answer

A	B	C	$A+B'$	$(BC)'$	$(A+B') \bullet (BC)'$	$A \oplus B'$	F
0	0	0	1	1	1	1	1
0	0	1	1	1	1	1	1
0	1	0	0	1	0	0	1
0	1	1	0	0	0	0	1
1	0	0	1	1	1	0	0
1	0	1	1	1	1	0	0
1	1	0	1	1	1	1	1
1	1	1	1	0	0	1	0

4. Use Boolean algebra to convert the function

$$F = ((A+B') \bullet (BC)') \odot (A \oplus B')$$

to its sum-of-products format. (3)

Answer

$F = ((A+B') \bullet (BC)') \odot (A \oplus B')$

$= ((A+B') \bullet (B'+C')) \odot (A \odot B)$

$= ((A+B') \bullet (B'+C')) \odot (AB+A'B')$

$= (AB' + AC' + B' + B'C') \odot (AB+A'B')$

$= [(AB' + AC' + B' + B'C') \bullet (AB+A'B')] +$
$[(AB' + AC' + B' + B'C')' \bullet (AB+A'B')']$

$= [($~~$AB'AB$~~ $+ AC'AB +$ ~~$B'AB$~~ $+$ ~~$B'C'AB$~~ $+$ ~~$AB'A'B'$~~ $+$ ~~$AC'A'B'$~~ $+ B'A'B' + B'C'A'B')] +$
$[(AB' + AC' + B' + B'C')' \bullet (AB+A'B')']$

$= \{(ABC' + A'B' + A'B'C')\} +$
$[(AB' + AC' + B' + B'C')' \bullet (AB+A'B')']$

$= \{ '' \} +$
$[(AB')' \bullet (AC')' \bullet (B')' \bullet (B'C')' \bullet (AB)' \bullet (A'B')']$

$= \{ '' \} +$
$[(A'+B) \bullet (A'+C) \bullet (B) \bullet (B+C) \bullet (A'+B') \bullet (A+B)]$

$= \{ '' \} +$
~~$A'A'BBA'A$~~ $+ A'A'BBA'B +$ ~~$A'A'BBB'A$~~ $+$ ~~$A'A'BBB'B$~~ $+$
~~$A'A'BCA'A$~~ $+ A'A'BCA'B +$ ~~$A'A'BCB'A$~~ $+$ ~~$A'A'BCB'B$~~ $+$
~~$A'CBBA'A$~~ $+ A'CBBA'B +$ ~~$A'CBBB'A$~~ $+$ ~~$A'CBBB'B$~~ $+$
~~$A'CBCA'A$~~ $+ A'CBCA'B +$ ~~$A'CBCB'A$~~ $+$ ~~$A'CBCB'B$~~ $+$

~~$BA'BBA'A$~~ $+ BA'BBA'B +$ ~~$BA'BBB'A$~~ $+$ ~~$BA'BBB'B$~~ $+$
~~$BA'BCA'A$~~ $+ BA'BCA'B +$ ~~$BA'BCB'A$~~ $+$ ~~$BA'BCB'B$~~ $+$
~~$BCBBA'A$~~ $+ BCBBA'B +$ ~~$BCBBB'A$~~ $+$ ~~$BCBBB'B$~~ $+$
~~$BCBCA'A$~~ $+ BCBCA'B +$ ~~$BCBCB'A$~~ $+$ ~~$BCBCB'B$~~ $+$

$= \{ '' \} + A'B + A'BC$

$= \{(ABC' + A'B' + A'B'C')\} + A'BC' + A'BC$

$= ABC' + A'B' + A'B'C' + A'BC' + A'BC$

$= A'B'C' + A'B'C + A'BC' + A'BC + ABC'$

5. Use Boolean algebra to simplify the following equation as much as possible and draw the circuit for it. (3)

$$F = ((A+B') \cdot (BC)') \odot (A \oplus B')$$

Answer

$F = (AB' + BC)' \oplus (A \odot B')$

$= A'B'C' + A'B'C + A'BC' + A'BC + ABC'$ from question 4

$= A'B'C' + A'B'C + A'BC' + A'BC + A'BC' + ABC'$

$= A'B'(C' + C) + A'B(C' + C) + A'BC' + ABC'$

$= A'B' + A'B + (A+A')BC'$

$= A'(B' + B) + BC'$

$= A' + BC'$