

CS/EE 120A

Homework #1 Given 1/18/01. Due 1/25/01

20

1. Any function can be implemented directly as specified or as its inverted form with a NOT gate added at the final output. Assume that the circuit size is proportional to only the number of AND gates and OR gates, i.e. ignore the number of NOT gates in determining the circuit size. Determine which form of the function (the inverted or un-inverted) will result in a smaller circuit size for the following function. Give your reason and specify how many AND and OR gates are needed to implement the smaller circuit. (5)

$$F = x'y'z' + x'y'z + xy'z + xy'z' + xyz$$

Answer

Convert the equation to the truth table

x	y	z	F	F'
0	0	0	1	0
0	0	1	1	0
0	1	0	0	1
0	1	1	0	1
1	0	0	1	0
1	0	1	1	0
1	1	0	0	1
1	1	1	1	0

From the truth table, we see that using the **inverted** form (F') will result in a smaller circuit size because it requires only 3 minterms while the un-inverted form (F) requires 5 minterms. 3 minterms with 3 variables require 8 3-input AND and 3-input OR gates, whereas 5 minterms require 14 gates. This is also true if we use multiple input AND and OR gates; F requires 6 gates whereas F' requires 4 gates.

Note that converting from sum of products to product of sums does not work because it will produce an inverted circuit that is the same size as the original.

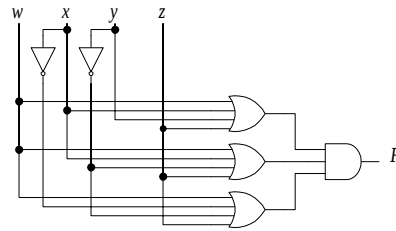
- ☞ 1 point for saying the inverted form is smaller.
- ☞ 4 points for giving the correct reason.

2. Using AND, OR, and NOT gates, draw the circuit that implements the following function: (5)

$$F(w, x, y, z) = \prod(0, 2, 6)$$

Answer

$$\begin{aligned} F(w, x, y, z) &= \prod(0, 2, 6) \\ &= M_0 + M_2 + M_6 \\ &= (w + x + y + z) (w + x + y' + z) (w + x' + y' + z) \end{aligned}$$



3. Convert the function $F = x'y'z + xy'z' + xy'z$ to:

(4)

- a) its standard form using the minimum number of logical operators.
- b) its nonstandard form using the minimum number of logical operators.

Answer:

a) (2 points)

$$\begin{aligned} F &= x'y'z + xy'z' + xy'z \\ &= (x' + x)y'z + xy'(z+z') \\ &= y'z + xy' \end{aligned}$$

b) (2 points)

$$\begin{aligned} F &= x'y'z + xy'z' + xy'z \\ &= (x' + x)y'z + xy'(z+z') \\ &= y'z + xy' \\ &= y'(x + z) \end{aligned}$$

4. Perform the following calculations using 5-bit 2's complement binary arithmetic. Point out if there is an overflow error. Show your work. (6)

a)

$$\begin{array}{r} 01110 \\ + 01010 \\ \hline \end{array}$$

Answer

$$\begin{array}{r} 11000 \\ \hline \end{array} \quad \text{overflow error}$$

b)

$$\begin{array}{r} 01110 \\ \times 11010 \\ \hline \end{array}$$

Answer

$$\begin{array}{r} 0000000000 \\ 0000011110 \\ 0000000000 \\ 00011110 \\ + 110010 \\ \hline 1110101100 \end{array}$$

c)

$$\begin{array}{r} 01101 \overline{)00100111} \end{array}$$

Answer

$$\begin{array}{r} 11 \\ 01101 \overline{)00100111} \\ \underline{01101} \\ 001101 \\ \underline{01101} \\ 0 \end{array}$$

👉 2 points for each part.