Problem Set 3
Due via turn-in on Friday, March 3, 2006

Question 1. [40 points]
There are a series of people standing in a line, from left to right. We'd like to reason about them using predicate logic. We have two predicates: **End(x)** which is true only if person x is standing at one of the two ends of the line, and **LeftOf(x,y)** which is true if person x is standing to the left of person y.

part a. [15 points] Construct a knowledge base that represents the following axioms about the world. Each should take one predicate logic sentence. Feel free to use equality if necessary.

1. LeftOf is a transitive operation.[1]
2. If a person is on the end, then everyone is on one side of him (either left or right).
3. Only two people can be at the end of the line.
4. A person can't be both to the left and to the right of the same person.

part b. [5 points] Add statements to reflect the following observations about our line.

1. Alice is standing to the left of Bob.
2. Bob is standing to the left of Charles.
3. Diane is not standing to the left of Bob.
4. Diane is on one end of the line.

part c. [20 points] Prove that Charles is standing to the left of Diane (*i.e.* show that is it entailed from the statements in parts a and b). Use proof by contradiction and unit resolution as shown in class. Show your steps.

[1] Transitivity means that relationship is consistent with an ordering. As an example, if Alice is standing to the left of Bob and Bob is standing to the left of Charles, we can conclude that Alice is standing to the left of Charles.

Question 2. [40 points]
part a. [10 points] You are working in *Tart & Tasty*, an applesauce processing plant. Before squishing the apples into applesauce, they need to be able to tell whether there is a worm inside (presumably so that they can toss the apple out, but it probably isn't worth asking). In an effort to automate the process, *Tart & Tasty* has tasked a mechanical design team to come up with a machine that will detect worms inside apples. The team has worked hard and come up with a machine that has a false positive rate of 0.1% (that is only one out of every thousand apples that are good will be reported as bad) and a false negative rate of 0.01% (that is only one out of every ten thousand apples that are bad will be reported as good). They are very pleased!

Statistics show that one out of every 50,000 apples has a worm in it. The company wants to know, "If the machine reports that an apple has a worm, what is the probability that it actually has a worm?" Calculate this probability and show your steps.

part b. [10 points] The company is disappointed and contracts a different external team to construct a new different detector. The new team comes up with a design so radically different that its errors are complete independent from those of the first detector. However, it still has the same false positive and false negative rates. *Tart & Tasty* decides that the best thing to do would be to use both detectors. If either detector flags an apple as "bad," the apple will be thrown out. What is the probability that an apple will be bad if it is thrown out by this combined machine?

part c. [10 points] After seeing the results of your calculations above, the company decides that, actually, it would be better to toss the apple only if *both* detectors report that it is bad. What is the probability now that an apple will be bad if it is thrown out?

part d. [10 points] Why might the detector in part c actually be worse than the detector in part b? Demonstrate this numerically (with a probability calculation or two).