Problem 1: Find the Maclaurin series of \( f(x) = \frac{1}{(1-x)(1-2x)} \).

First, expand with partial fractions. Then assemble the Maclaurin series from the pieces.

\[
\frac{A}{1-x} + \frac{B}{1-2x} = \frac{1}{(1-x)(1-2x)}
\]

\[A(1-2x) + B(1-x) = 1\]

\[A = -1 \quad (x = 1)\]

\[B = 2 \quad \left( x = \frac{1}{2} \right)\]

\[
\frac{1}{(1-x)(1-2x)} = \frac{2}{1-2x} - \frac{1}{1-x} = 2 \sum_{n=0}^{\infty} (2x)^n - \sum_{n=0}^{\infty} x^n = \sum_{n=0}^{\infty} (2^{n+1} - 1)x^n
\]

Problem 2: Show that \( \sum_{n=0}^{\infty} n^n x^n \) diverges for all \( x \neq 0 \).

If \( x \neq 0 \), then

\[
L = \lim_{n \to \infty} \sqrt[n]{n^n x^n} = \lim_{n \to \infty} n|x| = \infty
\]

and the series diverges by the root test.
Problem 3: Find the radius of convergence for \( \sum_{n=0}^{\infty} \frac{x^{2n}}{3^n} \).

\[
L = \lim_{n \to \infty} \sqrt[n]{\left| \frac{x^{2n}}{3^n} \right|} = \lim_{n \to \infty} \frac{x^2}{3} = \frac{x^2}{3}
\]

Since \( L < 1 \) is required for convergence (and \( L > 1 \) for divergence) by the root test, the radius of convergence is given by \( L = \frac{R^2}{3} = 1 \) or \( R = \sqrt{3} \).