Quiz 7 - Tuesday

Name: ___________________________  ID: ___________  Section: ___

You have 15 minutes to complete this quiz. You must show your work to receive credit. There are more problems on the back.

**Problem 1:** Evaluate the sum: \( \sum_{n=0}^{\infty} e^{-n} \)

\[
\sum_{n=0}^{\infty} e^{-n} = \frac{1}{1 - e^{-1}} = \frac{e}{e - 1}
\]

**Problem 2:** Determine the convergence or divergence of the series

\( \sum_{n=4}^{\infty} \frac{\ln n}{n^2 - 3n} \).

One solution is to let \( a_n = \frac{\ln n}{n^2 - 3n} \) and \( b_n = n^{-3/2} \).

\[
\lim_{n \to \infty} \frac{a_n}{b_n} = \lim_{n \to \infty} \frac{\sqrt{n} \ln n}{n - 3} = \lim_{n \to \infty} \frac{\ln n}{\sqrt{2n}} + \lim_{n \to \infty} \frac{1}{\sqrt{n}} = \lim_{n \to \infty} \frac{\ln n}{\sqrt{2n}} + \lim_{n \to \infty} \frac{1}{\sqrt{n}} = \lim_{n \to \infty} \frac{\ln n}{\sqrt{2n}} + \lim_{n \to \infty} \frac{1}{\sqrt{n}} = 0
\]

The series \( \sum_{n=1}^{\infty} n^{-3/2} \) is a convergent \( p \)-series. Since the limit is 0 and \( \sum_{n=1}^{\infty} n^{-3/2} \) converges, the series \( \sum_{n=4}^{\infty} \frac{\ln n}{n^2 - 3n} \) must also converge.
Problem 3: Determine whether the series $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$ converges absolutely, conditionally, or not at all.

Since $n$ and $\ln n$ are both increasing functions of $n$, $\frac{1}{n \ln n}$ is a decreasing function of $n$. Alternating series with decreasing terms always converge. On the other hand, we can use the integral test

$$\int \frac{1}{n \ln n} \, dn = \ln(\ln n) + C \quad \lim_{n \to \infty} \ln(\ln n) = \infty$$

to conclude that $\sum_{n=2}^{\infty} \frac{|(-1)^n|}{n \ln n}$ diverges. Thus, the series in question converges conditionally.