Problem 1: Give an example of a divergent sequence \( \{a_n\} \) such that \( \lim_{n \to \infty} |a_n| \) converges. Justify that the sequence \( \{a_n\} \) is divergent and that \( \lim_{n \to \infty} |a_n| \) converges.

One example is \( a_n = (-1)^n \). The sequence fails to converge since it alternates between 1 and \(-1\) rather than converging to a limit. On the other hand, \( \lim_{n \to \infty} |(-1)^n| = 1 \).

Problem 2: Evaluate the sum: \( \sum_{n=0}^{\infty} \frac{8 + 2^n}{5^n} \)

\[
\sum_{n=0}^{\infty} \frac{8 + 2^n}{5^n} = 8 \sum_{n=0}^{\infty} \left( \frac{1}{5} \right)^n + \sum_{n=0}^{\infty} \left( \frac{2}{5} \right)^n = \frac{8}{1 - \frac{1}{5}} + \frac{1}{1 - \frac{2}{5}} = 10 + \frac{5}{3} = \frac{35}{3}
\]
Problem 3: Determine whether the series \( \sum_{n=1}^{\infty} \frac{n}{10n + 12} \) converges or diverges. Justify your answer.

Since \( \lim_{n \to \infty} \frac{n}{10n + 12} = \frac{1}{10} \), the terms do not tend to zero and so the series must diverge.