Problem 1

Assuming nearly uniform, but heavy traffic, show that in general it is impossible to prescribe the traffic flow at the entrance to a semi-infinite highway. (An intuitive explanation for why this is so is sufficient; no algebra is required.) In this situation what might happen to cars waiting to enter the highway?

Mathematically, this is because in heavy traffic the traffic waves travel backwards. Information travels from right to left, from the road’s interior to its entrance. Cars waiting to enter will be slowed down, possibly even forced to stop.
Problem 2

Given the function $\hat{u}(\rho)$, explain how to obtain the function $u(x, t)$.

Solve the PDE $\frac{\partial \rho}{\partial t}(x, t) + \frac{\partial}{\partial x}(\hat{u}(\rho(x, t))\rho(x, t)) = 0$ for $\rho(x, t)$, subject to initial conditions $\rho(x, 0)$ and boundary conditions for $\rho(x_{\text{bound}}, t)$ if there are boundaries $x_{\text{bound}}$. Finally, $u(x, t) = \hat{u}(\rho(x, t))$. 