Problem 1
This group work activity is based around exploring the second order ordinary differential equation
\[ tx'' + x' + tx = \frac{1}{4} \sin t. \]
Write this second order ODE as a first order ODE of the form \( z' = f(z, t) \) by introducing a new variable \( v = x' \). Note that \( z \) in our case will need to be the vector \( \langle x, v \rangle \). Find \( f(z, t) \).

Problem 2
Write a Matlab function which takes two arguments and computes \( f(z, t) \) (note that the argument \( z \) and the result will be vectors). You are not required to use Matlab; you may use another language if you prefer. You will need to be able to generate line plots.

Problem 3
Plot the vector valued function \( f((\cos t, -\sin t), t) \) for \( t \in [0, 4\pi] \) using the Matlab function you wrote. Plot the function at around hundred or so sample points. (Note that you will have difficulty at \( t = 0 \); one simple solution is to plot it at a tiny value like \( t = 10^{-10} \) instead.)

Problem 4
When solving ODE’s numerically, we compute approximate values for \( z_n \approx z(t_n) \) at a large number \( N \) of sample times \( t_0 \ldots t_N \) in the interval \([0, T] \). The sample points are equally spaced (\( t_{n+1} = t_n + h \) for fixed \( h \), with \( t_0 = 0 \) and \( t_N = T \). The forward Euler method for approximating the solution to an ODE \( z' = f(z, t) \) is given by
\[
\begin{align*}
z_0 &= \langle x(0), x'(0) \rangle \\
z_{n+1} &= z_n + hf(t_n, z_n)
\end{align*}
\]
Compute \( z_0 \ldots z_N \) for \( N = 400, T = 20, x(0) = 1 \). (What is \( x'(0) \)?)

Problem 5
Generate a plot of the points \( (t_n, x(t_n)) \).

Problem 6
Approximate the maximum and minimum values of \( x(t) \) over the interval \([0, T] \).