Problem 1
Suggest a traffic following model of the form \( \dot{v}_k(t) = f(x_k, x_{k-1}, v_k, v_{k-1}) \) that would lead to the relationship \( \hat{u}(\rho) = u_{\text{max}} \left( 1 - \frac{\rho}{\rho_{\text{max}}} \right) \).

Problem 2
Consider the model \( \dot{v}_k(t) = -\lambda (x_k(t) - x_{k-1}(t) + d) \).

(a) What do \( \lambda \) and \( d \) represent?

(b) Why is \(-\lambda\) more reasonable than \( \lambda \)?

(c) Show that no density-velocity relationship \( \hat{u}(\rho) \) is implied by this model. (Hint: Show that \( \hat{u}(\rho) \) implies \( \dot{v}_k(t) = (v_k(t) - v_{k-1}(t))f(x_k(t) - x_{k-1}(t)) \) for some function \( f(x) \).)

(d) Do you think this model reasonably describes the behavior of drivers?

(e) Construct a traffic following model with a density-velocity relationship \( \hat{u}(\rho) \) such that drivers try to maintain a fixed following time. (For example, drivers always try to stay two seconds behind the car in front.) Is this always possible?

(f) Is the model \( \dot{v}_k(t) = \frac{v_k(t) - v_{k-1}(t)}{x_k(t) - x_{k-1}(t) + d} \) plausible?

Problem 3
Find the general solution to each of the differential equations below.

(a) \( x''' = x \)

(b) \( tx' + ax = 0 \)

(c) \( tx' + ax = b + ct \)

(d) \( t^2x'' + 4tx + 2x = 0 \)

(e) \( \frac{\partial f}{\partial x} \frac{\partial f}{\partial y} = 0 \)

(f) \( \frac{\partial^2 f}{\partial x \partial y} = 0 \)

(g) \( \left( \frac{\partial f}{\partial x} \right)^2 = \left( \frac{\partial f}{\partial y} \right)^2 \)

(h) \( \frac{\partial^2 f}{\partial x^2} = \frac{\partial^2 f}{\partial y^2} \)