Problem 1
A massless and frictionless pulley of radius $r$ centered at the origin has a rope of length $\ell$ draped over it. The position of the rope can be described by $x(t)$, the distance of the right end of the rope below the center of the pulley. The rope has total mass $m$, uniformly distributed along its length.

(a) What are the coordinates of the endpoints of the rope?

(b) What is the total gravitational potential energy ($\phi$) of the rope?

(c) The motion of the rope can be described by $\dot{x}(t)$. What is the total kinetic energy ($KE$) of the rope?

(d) What is the total energy of the rope?

(e) Derive a second order ODE that describes the motion of the system.

(f) Find the equilibria of the system. Are they stable?