$m\ddot{x} = -kx$ spring (1D)

$m\ddot{x}\dot{x} = -kx\dot{x}$

$\frac{1}{2}m(\dot{x})^2 = -\frac{1}{2}kx^2 + C$ integrate

$\underbrace{\frac{1}{2}mv^2}_{\text{kinetic energy}} + \underbrace{\frac{1}{2}kx^2}_{\text{potential energy}} = \underbrace{C}_{\text{total energy}}$ constant!

note that $\dot{C} = 0$, so total energy is <u>conserved</u>.

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let's go backwards

$\frac{1}{2}mv^2 + \underbrace{\phi}_{\text{potential energy}} = C$ $\phi(x)$

$mv\dot{v} + \frac{\partial\phi}{\partial x}\dot{x} = 0$

$mv\dot{v} = -\frac{\partial\phi}{\partial x}v$

$ma = -\frac{\partial\phi}{\partial x}$ $\Rightarrow$ $\boxed{f = -\frac{\partial\phi}{\partial x}}$ also true in 2D or 3D