$2x + 6y - 4z = 0 \rightarrow x + 3y - 2z = 0$

$x + 2y - z = 4 \rightarrow -y + z = 4 \rightarrow y - z = -4$

$-x + y + \cancel{3}z = 2 \rightarrow 4y - z = 2 \rightarrow 3z = 18 \rightarrow \boxed{z = 6}$

elimination

$y - z = -4 \rightarrow \boxed{y = z - 4 = 2}$

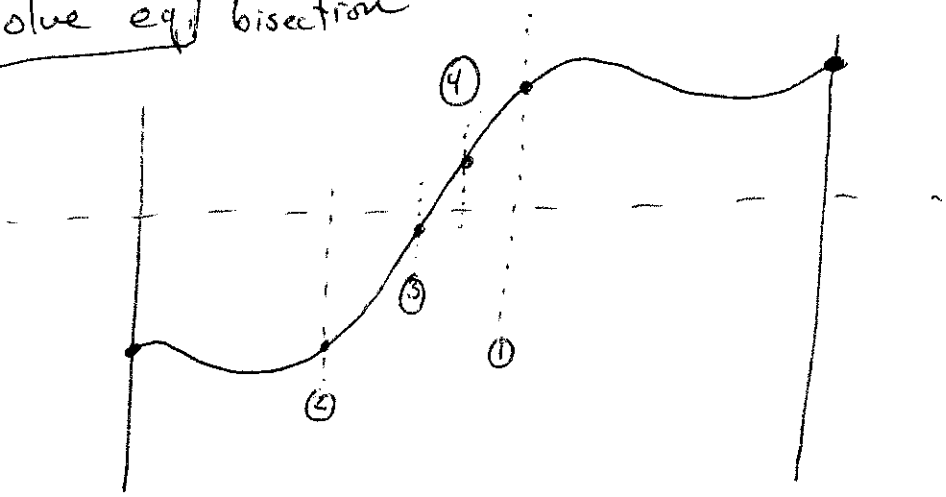
$x + 3y - 2z = 0$

$\boxed{x = -3y + 2z = -6 + 12 = 6}$

back-solve

1

solve eq: bisection



* guarantee
* require $f(a) < 0 < f(b)$ to start

---

* better than just div by 2?

$$\boxed{f(\overset{x_0+h}{x}) \approx f(x_0) + h f'(x_0)} + \frac{1}{2} h^2 f''(x_0) + \frac{1}{6} h^3 f'''(x_0) + \ldots$$

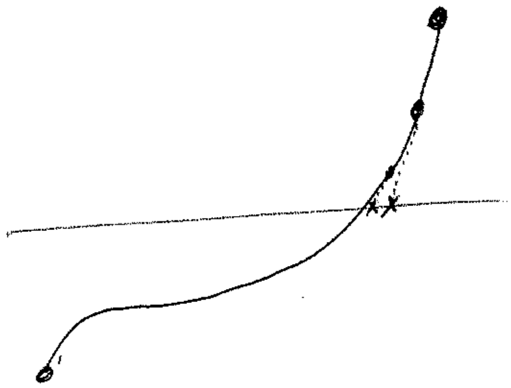
$f(x) = 0 \quad \Rightarrow \quad 0 = f(x_0 + h) = f(x_0) + h f'(x_0)$

guess correction

$$h = -\frac{f(x_0)}{f'(x_0)}$$

$$\boxed{x_1 \leftarrow x_0 - \frac{f(x_0)}{f'(x_0)}}$$

Newton's method.



* start with guess
* iterate until done
* converges very rapidly if close enough
* no guarantee

Can combine Newton with bisection

System. $\vec{f}(\vec{x}) = \vec{0}$ n dofs, n eqs.

$$\vec{f}(\vec{x}+\vec{h}) \approx \underbrace{\vec{f}(\vec{x})}_{\text{vec}} + \underbrace{(\nabla \vec{f})}_{\substack{\text{mat} \\ \text{"jacobian"}}} \underbrace{\vec{h}}_{\text{vec}}$$

$\rightarrow h = -(\nabla f)^{-1} f$

$\vec{h} = -\nabla f(x)^{-1} f(x)$

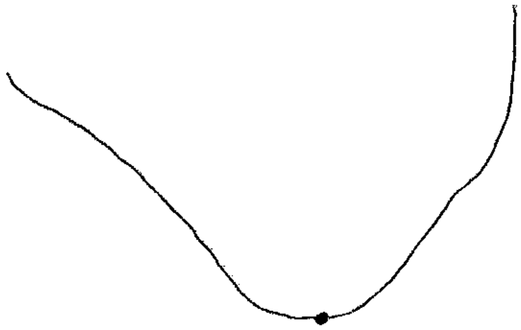
$x \leftarrow x - \nabla f(x)^{-1} f(x)$ Newton-Raphson

* solve linear system $(\nabla f)h = -f$
  - $\rightarrow$ do not compute inverse!

Optimization

$\min_x f(x)$

(or max)



$f'(x) = 0 \Rightarrow$ solve equation!

$\min_{x,y} f(x,y)$

$\Rightarrow \frac{\partial f}{\partial x} = 0 \quad \frac{\partial f}{\partial y} = 0$

if not, say $\frac{\partial f}{\partial x}(x,y) < 0$ then for small ε, $\frac{\partial f}{\partial x}(x,y) \approx \frac{f(x+\varepsilon,y) - f(x,y)}{\varepsilon} < 0$

$\rightarrow f(x+\varepsilon,y) < f(x,y) \rightarrow (x,y)$ not min

$\frac{\partial f}{\partial x}(x,y) > 0$ use $\frac{f(x,y) - f(x-\varepsilon,y)}{\varepsilon} > 0$ instead

$\nabla f(\vec{x}) = \vec{0} \Rightarrow n$ eq. n unkn.

$\Rightarrow$ Newton's method