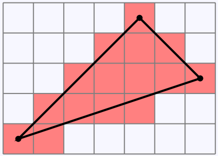
Line Rasterization

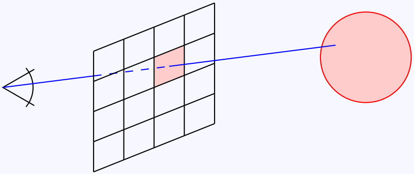
University of California Riverside

Raster Image

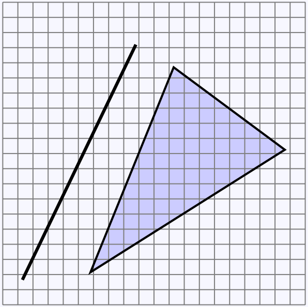
- Object oriented
  - for each object...



- Image oriented
  - for each pixel...



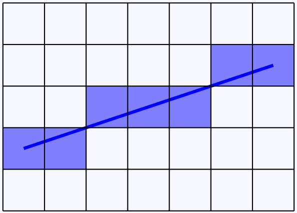
What is rasterization?



Rasterization is the process of determining which pixels are "covered" by the primitive

Rasterization

- In: 2D primitives (floating point)
- Out: covered pixels (integer)
- Must be fast (called **many times**)
- Visually pleasing
  - lines have constant width
  - lines have no gaps



DDA algorithm for lines

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- Plot line $y = mx + b$
- For each x:
  - $y = mx + b$
  - turn on pixel $(x, \mathrm{round}(y))$

DDA algorithm for lines

- Assume $|m| \leq 1$
- March from left to right

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$$\begin{aligned} y_{i+1} &= mx_{i+1} + b \\ &= m(x_i + 1) + b \\ &= y_i + m \end{aligned}$$

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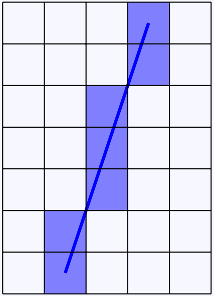
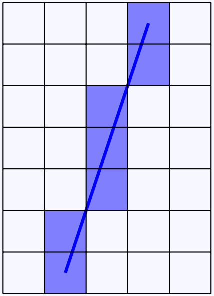
  - turn on pixel $(x_i, \text{round}(y_i))$

DDA algorithm for lines



- What if $|m| > 1$?

DDA algorithm for lines



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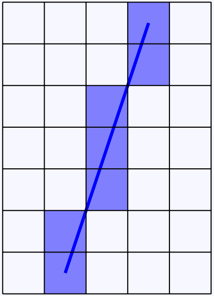

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- round(y) may skip an integer
    - gap in the line

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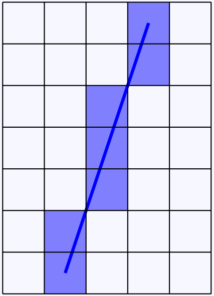

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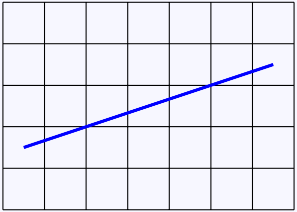
  - gap in the line
- Swap the roles of x and y
  - Loop over y, compute and round x

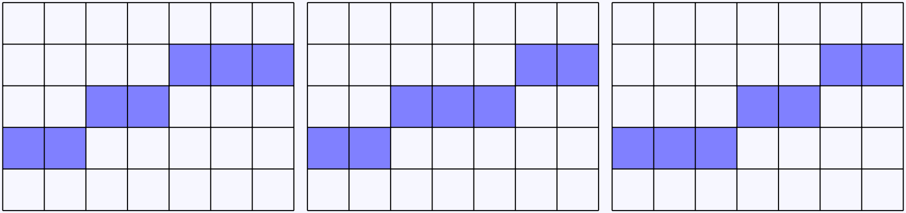
DDA algorithm for lines - limitations

- Must round for each pixel
  - very slow
- Only use ops: $+, -, \times$
  - Even better: $+, -$

Rasterization choices

- Thin, no gaps
- Still have choices

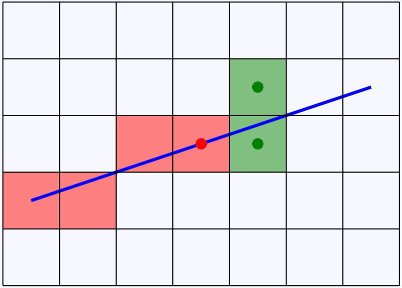




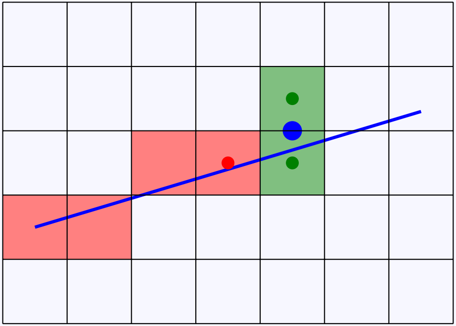
Midpoint algorithm

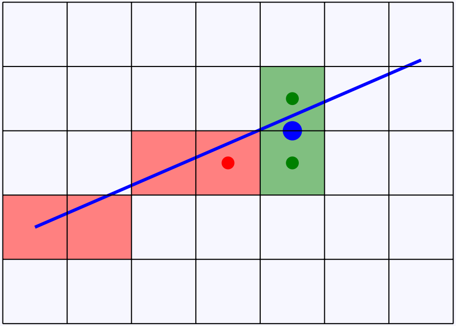
- Assume $0 \leq m \leq 1$
- Move from left to right
- Choose between $(x+1, y)$ and $(x+1, y+1)$

$y = y_0$
for $x = x_0, \ldots, x_1$ **do**
    draw(x, y)
    if $\langle$condition$\rangle$ **then**
        $y \leftarrow y + 1$



Check midpoint location





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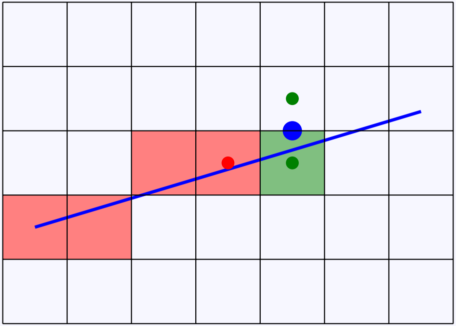
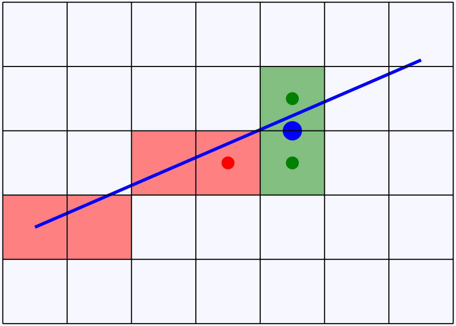

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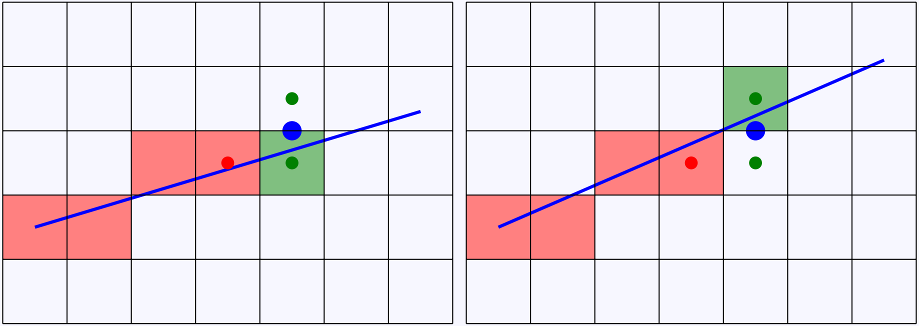
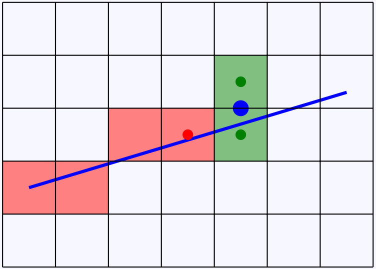


Criterion

Implicit line equation:

$$f(\mathbf{x}) = \mathbf{n} \cdot (\mathbf{x} - \mathbf{x}_0) = 0$$



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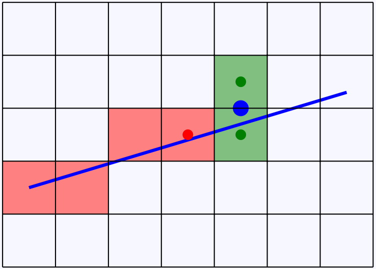
Evaluate f at midpoint:

$$f\left(x+1, y+\frac{1}{2}\right) \overset{?}{<} 0$$



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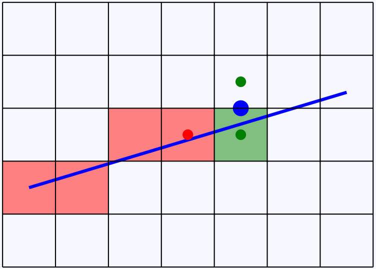
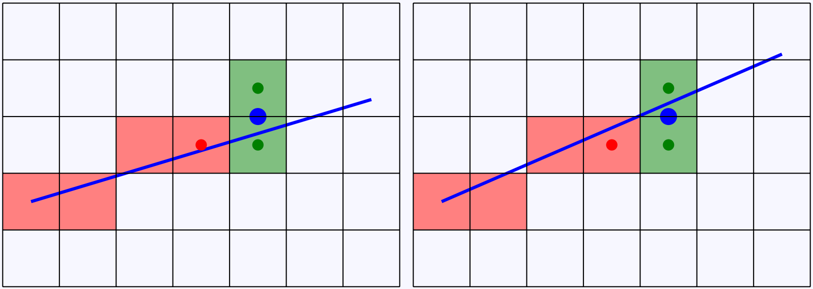


Midpoint algorithm ($0 \le m \le 1$)

$y \leftarrow y_0$
for $x = x_0, \ldots, x_1$ **do**
    $\text{draw}(x, y)$
    if $f(x+1, y+\frac{1}{2}) < 0$ **then**
        $y \leftarrow y + 1$



Efficiency: incremental update

- Compute initial $f(x, y)$
- Compute next by updating previous
- Update with *one* addition

$$f(x, y) = (y_0 - y_1)x + (x_1 - x_0)y + (x_0 y_1 - x_1 y_0)$$

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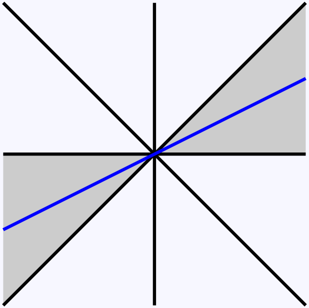
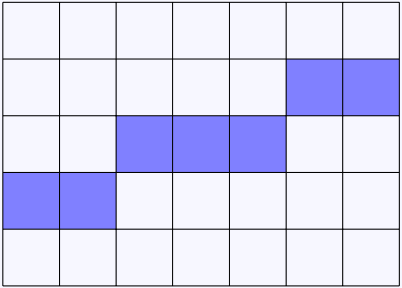
$$f(x+1, y+1) = f(x, y) + (y_0 - y_1) + (x_1 - x_0)$$

Efficiency: incremental update

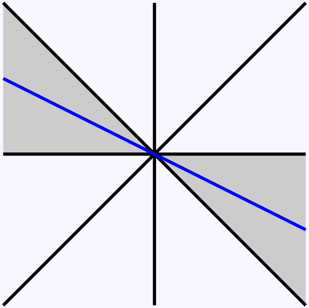
$y \leftarrow y_0$
$d \leftarrow f(x_0 + 1, y_0 + \frac{1}{2})$
for $x = x_0, \ldots, x_1$ **do**
    draw(x, y)
    if $d < 0$ **then**
        $y \leftarrow y + 1$
        $d \leftarrow d + (y_0 - y_1) + (x_1 - x_0)$
    else
        $d \leftarrow d + (y_0 - y_1)$

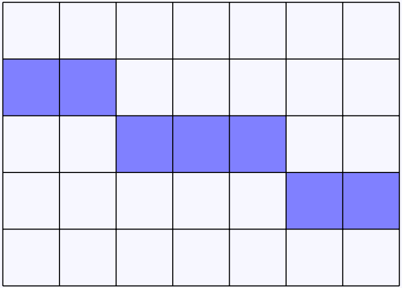
Other cases: $0 \leq m \leq 1$





Other cases: $-1 \leq m \leq 0$





Other cases: $|m| > 1$