

$$\vec{u} \times \vec{v} = \vec{w}$$

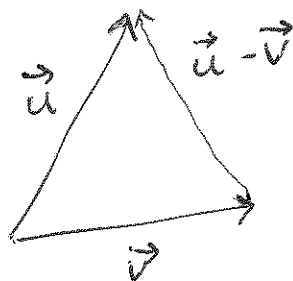
$$\|\vec{w}\| = \|\vec{u}\| \|\vec{v}\| \sin \theta \quad *$$

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos \theta \quad \square$$

$$(\vec{v} \cdot \vec{u})^2 + \|\vec{u} \times \vec{v}\|^2 = \|\vec{u}\|^2 \|\vec{v}\|^2 \quad \odot \leftarrow \text{easy to verify}$$

$$\square + \odot \Rightarrow *$$

$\square = \text{law of cosines}$



$$u \cdot v = v \cdot u$$

$$u \times v = -v \times u$$

$$(au) \cdot v = a(u \cdot v)$$

$$(au) \times v = a(u \times v)$$

$$(u \cdot v) + (u \cdot w) = u \cdot (v+w)$$

$$(u \times v) + (u \times w) = u \times (v+w)$$

$$\vec{u} \cdot \vec{u} = \|\vec{u}\|^2$$

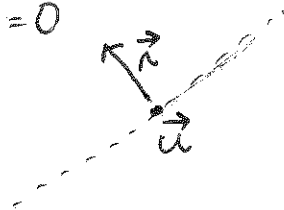
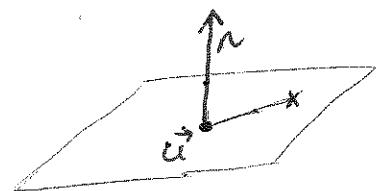
$$\vec{u} \times \vec{u} = \vec{0}$$

lines

$$\text{parametric: } f(t) = \vec{u} + t\vec{v}$$

$$(2D) \text{ implicit: } g(\vec{x}) = (\vec{x} - \vec{u}) \cdot \vec{n} = 0$$

$$(3D) \text{ implicit: } g(\vec{x}) = (\vec{x} - \vec{u}) \cdot \vec{n} = 0$$



Matrix

$$M = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

3x3

m rows
 n columns
 $m \times n$ matrix

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \pm \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} a_{11} \pm b_{11} & a_{12} \pm b_{12} \\ a_{21} \pm b_{21} & a_{22} \pm b_{22} \end{pmatrix}$$

$$c \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = \begin{pmatrix} c a_{11} & c a_{12} \\ c a_{21} & c a_{22} \end{pmatrix}$$

transpose: $\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}^T = \begin{pmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \end{pmatrix}$ $a_{ij} \leftrightarrow a_{ji}$

↙

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} u_1 \\ u_2 \end{pmatrix} = \begin{pmatrix} a_{11} u_1 + a_{12} u_2 \\ a_{21} u_1 + a_{22} u_2 \end{pmatrix} = \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

$$w_i = \sum_j a_{ij} u_j$$

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}$$

$$c_{ij} = \sum_k a_{ik} b_{kj}$$

$$A(Bv) = Cv = (AB)v$$

$$AB = C$$