CS 130, Midterm

Solutions

1	2	3	4	5	6	7	8	9	10	11	12

Read the entire exam before beginning. **Manage your time carefully.** This exam has 50 points; you need 40 to get full credit. Additional points are extra credit.

Short answer. For each question below, provide a brief 1-3 sentence explanation.

Problem 1 (2 points)

Give reasonable RGB values for (1) blue, (2) dark blue, and (3) light blue.

Blue is $(0, 0, 1)$. Dark blue is a mixture of blue with black: $(0, 0, 0.5)$. Light blue is a mixture with white: $(0.5, 0.5, 1)$.

Problem 2 (2 points)

Why might it be beneficial to cast multiple shadow rays to one light source?

This can be used for area lights to produce soft shadows. It can also be used for spatially-varying light sources, such as a computer screen.

Problem 3 (2 points)

Which of these dominates the cost of ray tracing when acceleration structures are not in use and why? (a) Computing rays to cast, (b) calculating surface normals, (c) shading computations, (d) intersections, or (e) texture mapping.

If there are p pixels, n objects, and l lights, ignoring reflections or transparency, the operations above will execute approximately (a) $p + pl$, (b) p, (c) p, (d) $pn + pnl$, (e) p times. Of these, intersections are the most expensive.

Problem 4 (2 points)

What are the barycentric coordinates of the center of an equilateral triangle?

$\alpha = \beta = \gamma = \frac{1}{3}$. They must be equal due to symmetry, and they must add to 1.

Problem 5 (2 points)

Given two vectors $\vec{u}$ and $\vec{w}$, construct a vector $\vec{q}$ orthogonal to both of them.

$\vec{q} = \vec{u} \times \vec{w}$

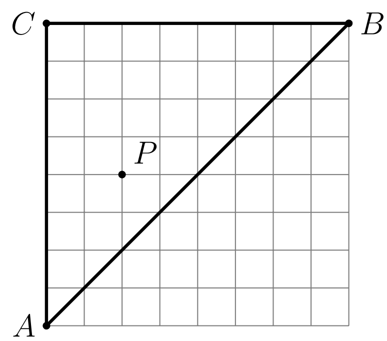
Problem 6 (2 points)

How might a viewer distinguish a rough asteroid model from a smooth asteroid with a bump map applied?

The silhouette in the first case will be rough, but it will be smooth in the second case.

Problem 7 (5 points)

Find the barycentric weights for the point P in the triangle below.



To do this, we need to compute the areas of the triangles, which we can then use to compute barycentric weights. Note that all but one of the triangles we need have an edge that is horizontal or vertical, so $A = \frac{1}{2}bh$ is easy to compute.

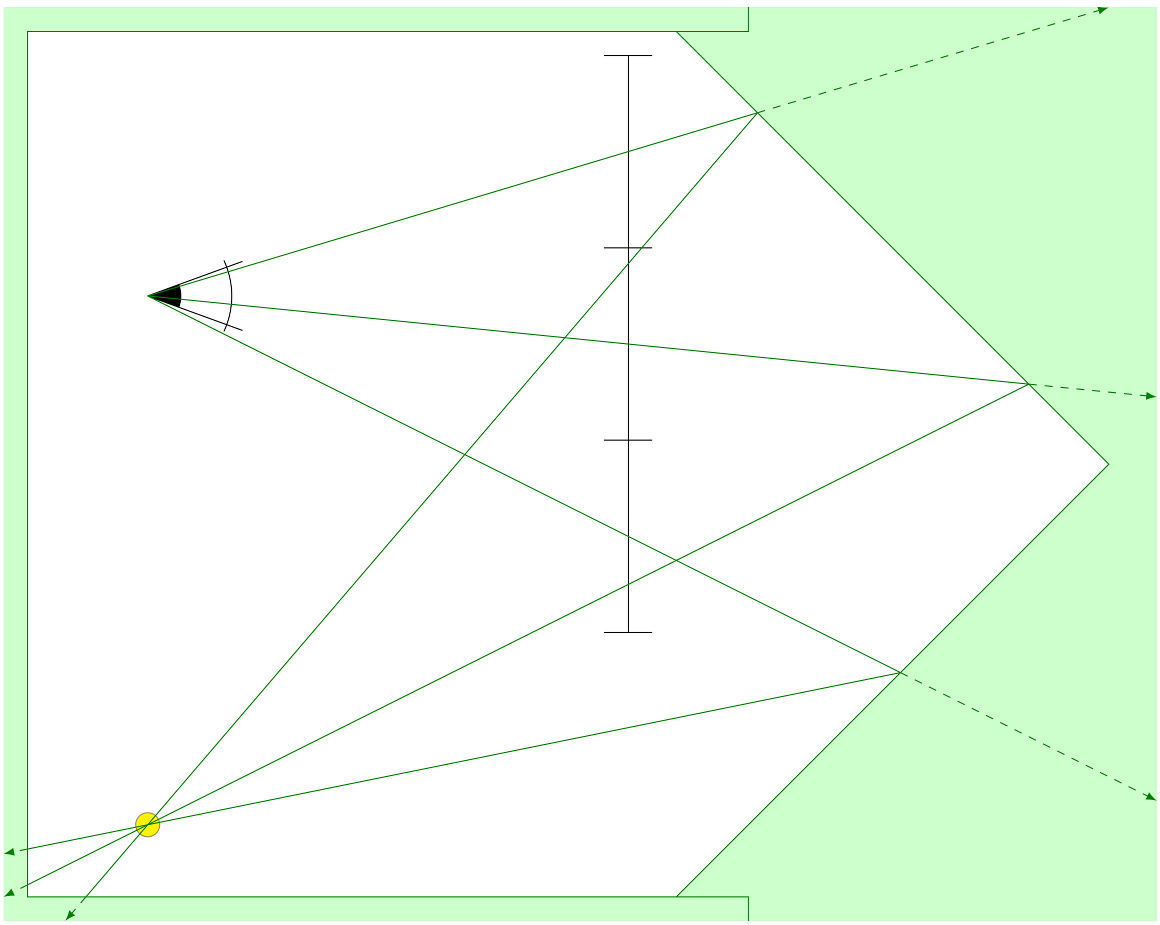
$$\text{area}(ABC) = 32 \qquad \text{area}(PBC) = 16 \qquad \text{area}(APC) = 8$$

$$\text{area}(ABP) = \text{area}(ABC) - \text{area}(PBC) - \text{area}(APC) = 8$$

$$\alpha = \frac{\text{area}(PBC)}{\text{area}(ABC)} = \frac{16}{32} = \frac{1}{2} \quad \beta = \frac{\text{area}(APC)}{\text{area}(ABC)} = \frac{8}{32} = \frac{1}{4} \quad \gamma = \frac{\text{area}(ABP)}{\text{area}(ABC)} = \frac{8}{32} = \frac{1}{4}$$

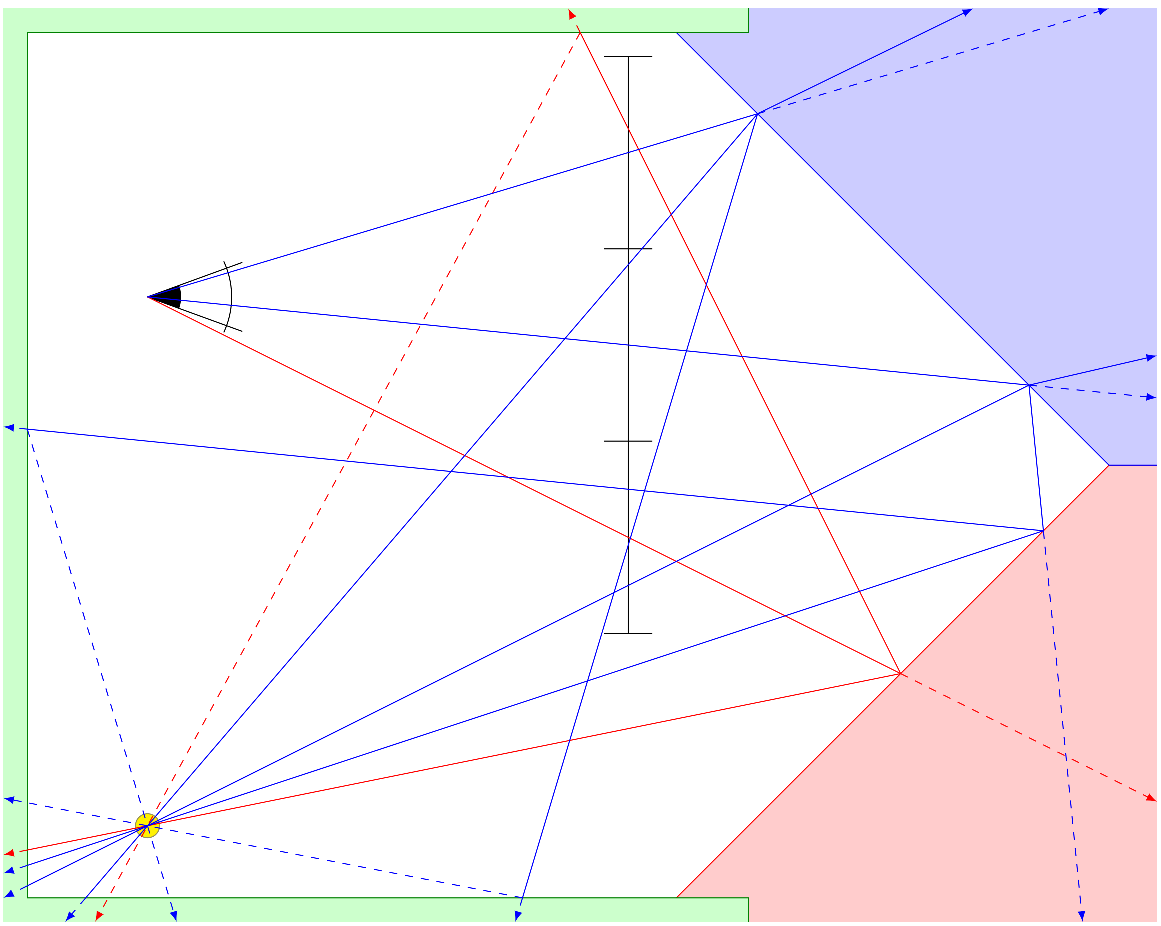
Problem 8 (10 points)

Below is a simple 2D raytracing setup. The 1D image has three pixels. The green object is made of wood. The yellow circle is a point light. Draw all of the rays that would be cast while raytracing this scene. (Tip: use the edge of a piece of paper or a pencil to draw rays; it helps.)



Problem 9 (10 points)

Below is a simple 2D raytracing setup. The 1D image has three pixels. The green object is made of wood. The blue object is made of glass (reflective and transparent). The red object is reflective. The yellow circle is a point light. Draw all of the rays that would be cast while raytracing this scene.



Problem 10 (4 points)

Let R be the ray with endpoint $(2, -3, 1)$ and direction $(0, 1, 0)$. Let S be the sphere centered at the origin with radius 3. Find all intersection *locations* between the sphere and the ray.

The ray is given by $x = 2$, $y = -3 + t$, $z = 1$. The sphere is given by $x^2 + y^2 + z^2 = 9$. Substituting in gives $4 + (t-3)^2 + 1 = 9$ or $(t-3)^2 = 4$ so that $t - 3 = \pm 2$. This gives two solutions $t = 1$ and $t = 5$. These correspond to the locations $(2, -2, 1)$ and $(2, 2, 1)$.

Problem 11 (5 points)

A surface (rather like a spiral staircase) is given by the parametric equations $x = r\cos\theta$, $y = r\sin\theta$, $z = \theta$. What is the normal direction at an arbitrary point on the surface? You may express your result as a function of (r, θ) or (x, y, z) as you prefer.

$$\begin{aligned}
\mathbf{w} &= \begin{pmatrix} r\cos\theta \\ r\sin\theta \\ \theta \end{pmatrix} \\
\mathbf{w}_r &= \begin{pmatrix} \cos\theta \\ \sin\theta \\ 0 \end{pmatrix} \\
\mathbf{w}_\theta &= \begin{pmatrix} -r\sin\theta \\ r\cos\theta \\ 1 \end{pmatrix} \\
\mathbf{w}_r \times \mathbf{w}_\theta &= \begin{pmatrix} (\sin\theta)1 - 0(r\cos\theta) \\ 0(-r\sin\theta) - (\cos\theta)1 \\ (\cos\theta)(r\cos\theta) - (\sin\theta)(-r\sin\theta) \end{pmatrix} = \begin{pmatrix} \sin\theta \\ -\cos\theta \\ r \end{pmatrix} \\
\|\mathbf{w}_r \times \mathbf{w}_\theta\|^2 &= (\sin\theta)^2 + (-\cos\theta)^2 + r^2 = r^2 + 1 \\
n &= \frac{\mathbf{w}_r \times \mathbf{w}_\theta}{\|\mathbf{w}_r \times \mathbf{w}_\theta\|} = \frac{1}{\sqrt{r^2+1}} \begin{pmatrix} \sin\theta \\ -\cos\theta \\ r \end{pmatrix} = \frac{1}{\sqrt{x^2+y^2+1}\sqrt{x^2+y^2}} \begin{pmatrix} y \\ -x \\ x^2+y^2 \end{pmatrix}
\end{aligned}$$

Problem 12 (4 points)

The operation $A \operatorname{xor} B$ consists of all points in A or B but not both. The operation is illustrated on two circles below. Fill in the case table for this operation in the table below.

A	I	I	O	O	E	L	E	L
B	E	L	E	L	I	I	O	O
$A \operatorname{xor} B$	L	E	E	L	L	E	E	L