

CS130 - LAB - Math review

Name: _____

SID: _____

Part 1: vectors, dot and cross product

1. Solve the given vector equation for x . Is there a solution? $3 \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + 4 \begin{pmatrix} 2 \\ 0 \\ x \end{pmatrix} = \begin{pmatrix} 11 \\ 6 \\ 17 \end{pmatrix}$.

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2. Solve the given vector equation for the scalar x . Is there a solution? $x \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + 4 \begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \\ 4 \end{pmatrix}$

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3. Calculate the cosine of the angle between the vectors $\begin{pmatrix} 2 \\ 4 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix}$.

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4. Given two vectors, \mathbf{u} and \mathbf{v} , explain the geometrical relationship between \mathbf{u} and \mathbf{v} for the following cases: (a) $\mathbf{u} \cdot \mathbf{v} = 0$, (b) $\mathbf{u} \cdot \mathbf{v} > 0$, and (c) $\mathbf{u} \cdot \mathbf{v} < 0$.

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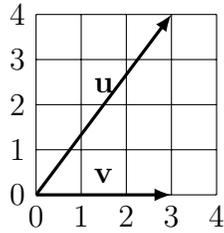
5. Calculate the cross product: $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$.

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6. Given the triangle with vertices $(0, 2, -1)$, $(2, 0, -1)$ and $(1, 0, 0)$, calculate the normal of the plane that contains the triangle.

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7. Calculate the vector that bisects the angle between the vectors \mathbf{u} and \mathbf{v} in the figure below.



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8. Calculate a vector \mathbf{w} in the same direction of the vector \mathbf{u} and that has the same length as the vector \mathbf{v} .

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Part 2: Matrices

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 5 \\ 3 & 7 & 19 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 5 & 2 \\ 1 & -3 \\ -1 & 1 \end{pmatrix} \quad \mathbf{u} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

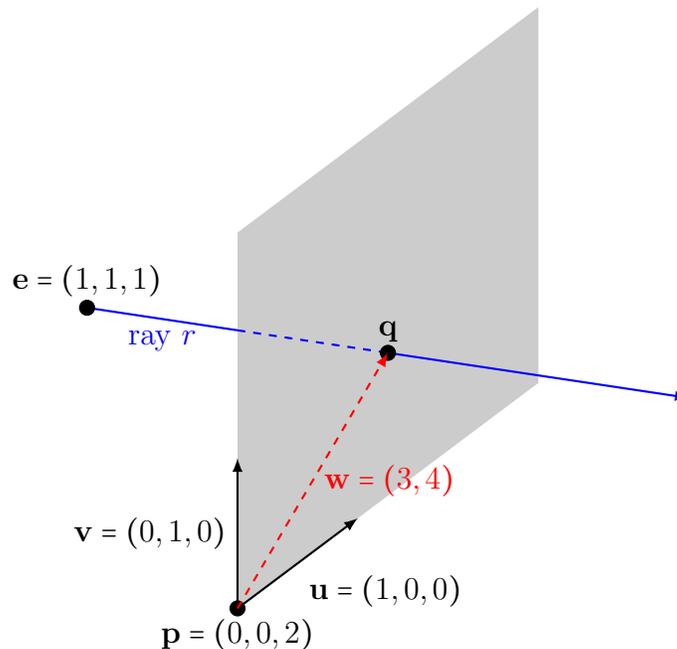
9. Calculate: (a) $\mathbf{A} + \mathbf{B}^T$, (b) \mathbf{AB} , and (c) $(\mathbf{AB})^{-1}$.

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10. Solve $(\mathbf{AB})\mathbf{x} = \mathbf{u}$ for \mathbf{x} . Show the following steps: (a) Isolate \mathbf{x} in the left-hand side of the equation. (b) Compute $(\mathbf{AB})^{-1}\mathbf{u}$ to find the values of \mathbf{x} .

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Part 3: Ray/plane intersection



11. Calculate the endpoint and direction (don't forget to normalize) of the ray r in the figure above.

- \mathbf{u} and \mathbf{v} are unit vectors that define the orientation of the shaded plane.
- The origin of the plane is at \mathbf{p} .
- The ray passes through the plane at the intersection point \mathbf{q} .
- The 2D vector \mathbf{w} is on the plane and goes from \mathbf{p} (plane origin) to \mathbf{q} .

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12. Consider a ray with endpoint \mathbf{e} and (unit) direction \mathbf{u} . Consider a plane with (unit) normal vector \mathbf{n} and with a point \mathbf{x}_0 located anywhere on the plane. The points on the ray are given by $R(t) = \mathbf{e} + t\mathbf{u}$ for $t \geq 0$. The points on the plane satisfy $P(\mathbf{x}) = (\mathbf{x} - \mathbf{x}_0) \cdot \mathbf{n} = 0$. Fill in the code for a C++ routine that returns whether the ray intersects the plane. You may assume that `dot(u,v)` and `cross(u,v)` routines exist and that `+`, `-`, `*`, `/` operators are overloaded with their standard meanings. Try to stick to C++ syntax as much as possible.

```
bool intersects(const vec3& e, const vec3& u, const vec3& n,  
               const vec3& x0)  
{
```

}

13. Consider a sphere with center \mathbf{c} and radius r , which is given by the formula $(\mathbf{x}-\mathbf{c})\cdot(\mathbf{x}-\mathbf{c}) = r^2$. Any point x that satisfies the sphere equation is on the sphere. Calculate t such that the corresponding point on the ray from the previous problem intersects the sphere. (You may use math or C++ notation for this one.)

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