Problem 1 (3 points)
A square with a letter (shown in the diagram labeled “orig” below) is transformed into each of the configurations (a)-(c). In each case, identify the type of transform and, if possible, find a $3 \times 3$ homogeneous transform matrix corresponding to it. In each case, identify the transform as a R=rotation, T=translation, S=uniform scale, X=none of these. R, S, and T can be combined. The most restrictive option should be chosen. Thus, a transform that can be accomplished by a combination of rotation and uniform scale should be described as R+S, not as X.

(a) R. \[
\begin{pmatrix}
0 & -1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 1 \\
\end{pmatrix}
\]
(b) T. \[
\begin{pmatrix}
1 & 0 & -1 \\
0 & 1 & -2 \\
0 & 0 & 1 \\
\end{pmatrix}
\]
(c) S+T. \[
\begin{pmatrix}
2 & 0 & -1 \\
0 & 2 & -1 \\
0 & 0 & 1 \\
\end{pmatrix}
\]
Problem 2 (1 points)

What does the $4 \times 4$ homogeneous transform matrix $M$ do?

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

This problem is actually a little tricky, since we have not discussed the effects of putting something other than 1 in the bottom right. Nevertheless, we can simply examine how it changes a point $(x, y, z)$.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ 2 \end{bmatrix} \equiv \begin{bmatrix} x \frac{1}{2} \\ y \frac{1}{2} \\ z \frac{1}{2} \\ 1 \end{bmatrix}$$

This matrix behaves as a uniform scale by $\frac{1}{2}$. Indeed, since scaling a homogeneous point makes no difference (since we will eventually divide off the $w$), scaling a transformation also makes no difference.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 2 \end{bmatrix} \equiv \begin{bmatrix} \frac{1}{2} & 0 & 0 & 0 \\ 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This is the form we would expect from a scale matrix.