CS130 - LAB - Math review

Name: ________________________________ SID: ____________

Part 1: vectors, dot and cross product

1. Solve the given vector equation for x. Is there a solution? $3\begin{pmatrix}1\\2\\-1\end{pmatrix} + 4\begin{pmatrix}2\\0\\x\end{pmatrix} = \begin{pmatrix}11\\6\\17\end{pmatrix}$.

■

2. Solve the given vector equation for the scalar x. Is there a solution? $x\begin{pmatrix}1\\2\\-1\end{pmatrix} + 4\begin{pmatrix}3\\4\\2\end{pmatrix} = \begin{pmatrix}-1\\0\\4\end{pmatrix}$

■

3. Calculate the cosine of the angle between the vectors $\begin{pmatrix}2\\4\\4\end{pmatrix}$ and $\begin{pmatrix}4\\3\\0\end{pmatrix}$.

$\blacksquare$

4. Given two vectors, $\mathbf{u}$ and $\mathbf{v}$, explain the geometrical relationship between $\mathbf{u}$ and $\mathbf{v}$ for the following cases: (a) $\mathbf{u} \cdot \mathbf{v} = 0$, (b) $\mathbf{u} \cdot \mathbf{v} > 0$, and (c) $\mathbf{u} \cdot \mathbf{v} < 0$.

$\blacksquare$

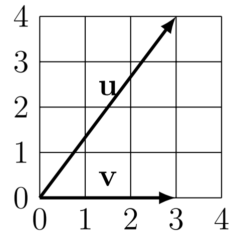
5. Calculate the cross product: $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$.

$\blacksquare$

6. Given the triangle with vertices $(0, 2, -1)$, $(2, 0, -1)$ and $(1, 0, 0)$, calculate the normal of the plane that contains the triangle.

$\blacksquare$

7. Calculate the vector that bisects the angle between the vectors $\mathbf{u}$ and $\mathbf{v}$ in the figure below.



$\blacksquare$

8. Calculate a vector $\mathbf{w}$ in the same direction of the vector $\mathbf{u}$ and that has the same length as the vector $\mathbf{v}$.

$\blacksquare$

Part 2: Matrices

$$\mathbf{A} = \begin{pmatrix} 1 & 2 & 5 \\ 3 & 7 & 19 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 5 & 2 \\ 1 & -3 \\ -1 & 1 \end{pmatrix} \qquad \mathbf{u} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$

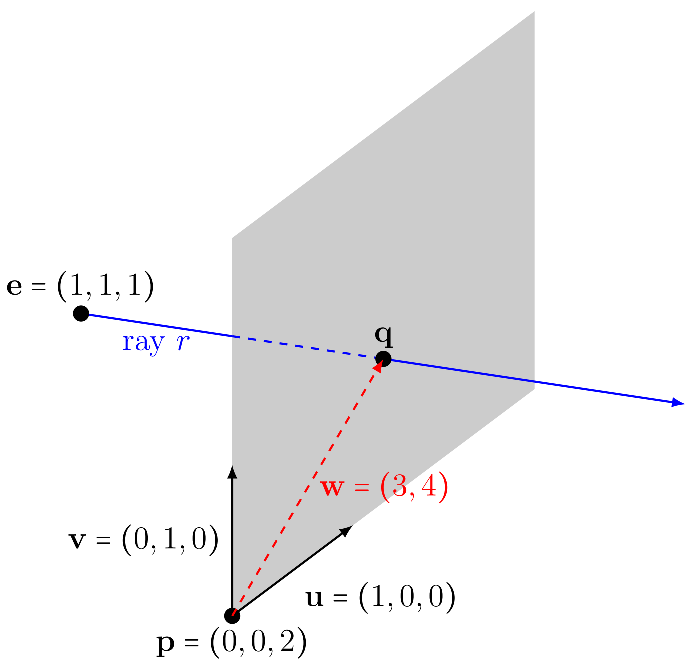
9. Calculate: (a) $\mathbf{A} + \mathbf{B}^T$, (b) $\mathbf{AB}$, and (c) $(\mathbf{AB})^{-1}$.

■

10. Solve $(\mathbf{AB})\mathbf{x} = \mathbf{u}$ for $\mathbf{x}$. Show the following steps: (a) Isolate $\mathbf{x}$ in the left-hand side of the equation. (b) Compute $(\mathbf{AB})^{-1}\mathbf{u}$ to find the values of $\mathbf{x}$.

■

Part 3: Ray/plane intersection



11. Calculate the endpoint and direction (don't forget to normalize) of the ray r in the figure above.

- **u** and **v** are unitary vectors that define the orientation of the shaded plane.
- The origin of the plane is at **p**.
- The ray passes through the plane at the intersection point **q**.
- The 2D vector **w** is on the plane and goes from **p** (plane origin) to **q**.

■

12. Consider a ray with endpoint **e** and (unitary) direction **u**. Consider a plane with (unitary) normal vector **n** and with a point $\mathbf{x}_0$ located anywhere on the plane. The points on the ray are given by $R(t) = \mathbf{e} + t\mathbf{u}$ for $t \geq 0$. The points on the plane satisfy $P(\mathbf{x}) = (\mathbf{x} - \mathbf{x}_0) \cdot \mathbf{n} = 0$. Fill in the code for a C++ routine that returns whether the ray intersects the plane. You may assume that `dot(u,v)` and `cross(u,v)` routines exist and that $+, -, *, /$ operators are overloaded with their standard meanings. Try to stick to C++ syntax as much as possible.

```
bool intersects(const vec3& e, const vec3& u, const vec3& n,
    const vec3& x0)
{




}
```

13. Consider a sphere with center **c** and radius r, which is given by the formula $(\mathbf{x}-\mathbf{c})\cdot(\mathbf{x}-\mathbf{c}) = r^2$. Any point x that satisfies the sphere equation is on the sphere. Calculate t such that the corresponding point on the ray from the previous problem intersects the sphere. (You may use math or C++ notation for this one.)

■