You will have 6 minutes to complete this quiz. No books, notes, or other aids are permitted.

One technique that is used to compute \( z = \frac{1}{a} \) is to start with an estimate \( \hat{z} \) for \( z \) and then use Newton’s method to compute a more accurate estimate.

**Problem 1 (1 Point)**

Let \( f(x) = \frac{1}{x} - a \). Show that the desired quantity \( z \) satisfies the equation \( f(z) = 0 \).

\[
\begin{align*}
f(z) &= \frac{1}{z} - a = a - a = 0
\end{align*}
\]

**Problem 2 (4 Points)**

Given an estimate \( w \) to \( z \) (so that \( f(w) \approx 0 \)), use Newton’s method to derive an update rule to compute a better estimate \( y \) from the original estimate \( w \). (Hint: your update rule should not require any operations beyond addition, subtraction, or multiplication.)

\[
\begin{align*}
y &= w + h \\
0 &= f(y) = f(w + h) \approx f(w) + f'(w)h \\
h &= -\frac{f(w)}{f'(w)} \\
y &= w - \frac{f(w)}{f'(w)} \\
f(x) &= \frac{1}{x} - a \\
f'(x) &= -\frac{1}{x^2} \\
y &= w - \frac{1}{w} - \frac{a}{w^2} = w + (w - aw^2) = w(2 - aw)
\end{align*}
\]