One technique that processors use to compute \( z = \sqrt{a} \) is to first compute \( x = \frac{1}{\sqrt{a}} \) and then multiply \( z = ax \).

**Problem 1 (1 Point)**

Show that \( f(x) = 0 \).

\[
f(x) = \frac{1}{x^2} - a = \frac{1}{\left(\frac{1}{\sqrt{a}}\right)^2} - a = \frac{1}{a} - a = a - a = 0
\]

**Problem 2 (4 Points)**

Given an estimate \( \bar{x} \) to \( x \) (so that \( f(\bar{x}) \approx 0 \)), use Newton’s method to derive an update rule to compute a better estimate \( \hat{x} \) from the original estimate \( \bar{x} \).

\[
\hat{x} = \bar{x} + y
\]

\[
0 = f(\hat{x}) = f(\bar{x} + y) \approx f(\bar{x}) + f'(\bar{x})y
\]

\[
y = -\frac{f(\bar{x})}{f'(\bar{x})}
\]

\[
\hat{x} = \bar{x} - \frac{f(\bar{x})}{f'(\bar{x})}
\]

\[
f(x) = \frac{1}{x^2} - a
\]

\[
f'(x) = -\frac{2}{x^3}
\]

\[
\hat{x} = \bar{x} - \frac{1}{\frac{2}{\bar{x}^2}} = \bar{x} + \frac{\bar{x} - a\bar{x}^3}{2} = \frac{1}{2} \bar{x}(3 - a\bar{x}^2)
\]