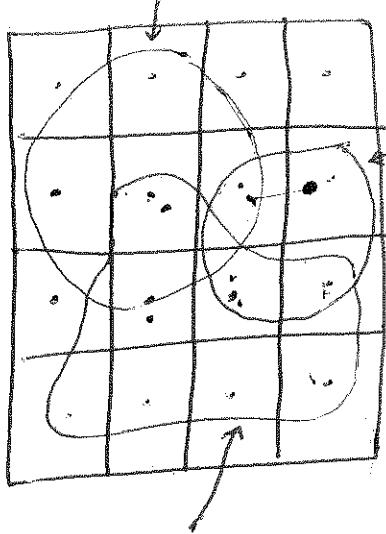


A dist = 4



C
dist = 5

Rasterize A

-	A/4	-	-
A/4	A/4	A/4	-
-	A/4	-	-
-	-	-	-

B

dist = 3
Rasterize B

-	A/4	-	-
A/4	B/3	A/4	-
-	B/3	B/3	B/3
B/3	B/3	B/3	B/3



-	A/4	-	-
A/4	B/3	A/4	C/5
-	B/3	B/3	B/3
B/3	B/3	B/3	B/3

Linear Operator

$$y = f(x) \quad x \xrightarrow{f} y$$

$$f(ax + by) = af(x) + bf(y)$$

↑ ↓
vectors

if x is a vector

$$y_i = \sum_k A_{ik} x_k \Rightarrow \vec{y} = A \vec{x}$$

$\xrightarrow{A} \xrightarrow{B} z$

best I can do...

↑
matrix

$$z_n = \sum_{ki} B_{ni} y_i = \sum_{ik} B_{ni} A_{ik} x_k = \underbrace{\sum_k \left(\sum_i B_{ni} A_{ik} \right) x_k}_{C_{nk}}$$

$$B(Ax) = Cx = (BA)x$$

transforms

2D

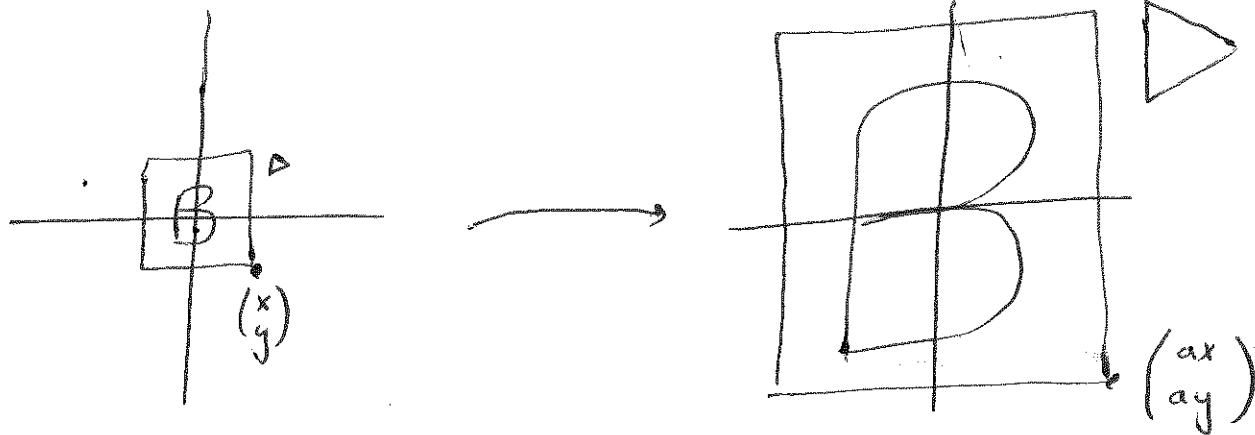
translation

rotation

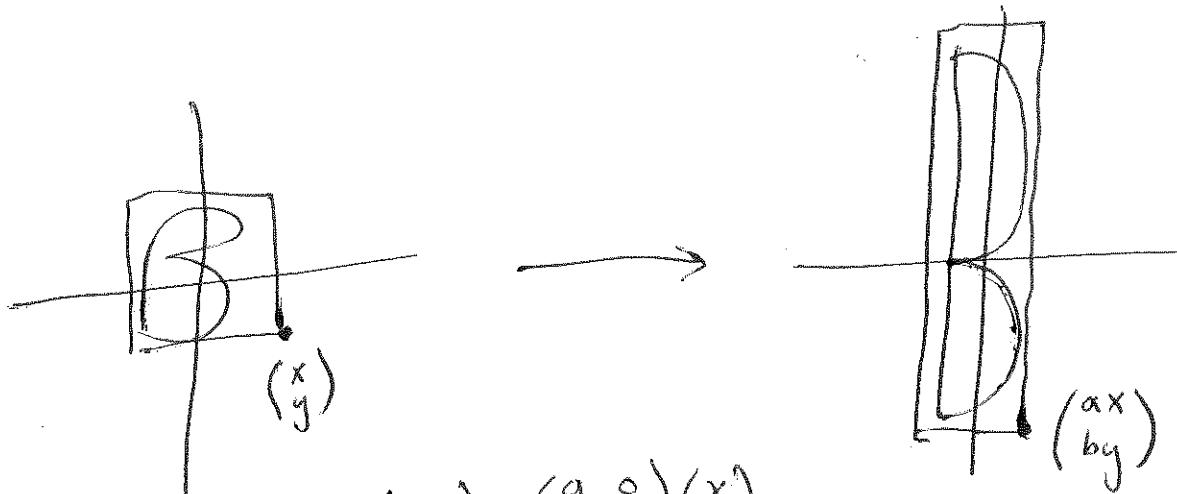
uniform scale

non-uniform scale

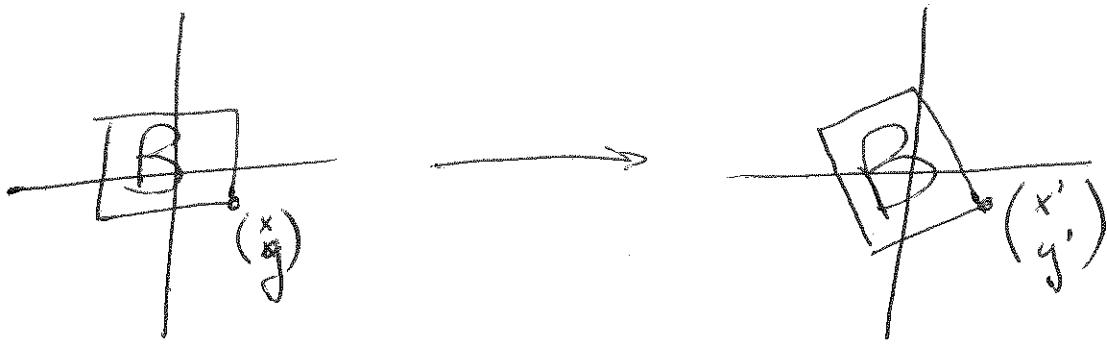
shear



$$\begin{pmatrix} ax \\ ay \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



$$\begin{pmatrix} ax \\ by \end{pmatrix} = \begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$



$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\left\| \begin{pmatrix} x \\ y \end{pmatrix} \right\|^2 = \left\| \begin{pmatrix} x' \\ y' \end{pmatrix} \right\|^2$$

$$\begin{aligned} (x')^2 + (y')^2 &= (x \cos \theta - y \sin \theta)^2 + (x \sin \theta + y \cos \theta)^2 \\ &= x^2 (\cos^2 \theta + \sin^2 \theta) + y^2 (\sin^2 \theta + \cos^2 \theta) \\ &\quad + \cancel{2xy(-\cos \theta \sin \theta + \sin \theta \cos \theta)} \\ &= x^2 + y^2 \end{aligned}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} \cdot \begin{pmatrix} x' \\ y' \end{pmatrix} = \|\vec{x}\| \|\vec{x}'\| \cos \theta = \|\vec{x}\|^2 \cos \theta$$

$$\begin{matrix} \uparrow & \uparrow \\ \vec{x} & \vec{x}' \end{matrix}$$

$$\begin{aligned} \begin{pmatrix} x \\ y \end{pmatrix} \cdot \begin{pmatrix} x \cos \theta - y \sin \theta \\ x \sin \theta + y \cos \theta \end{pmatrix} &= x^2 \cos \theta - xy \sin \theta + yx \sin \theta + y^2 \cos \theta \\ &= (x^2 + y^2) \cos \theta \end{aligned}$$

$$\begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}$$