Problem 1 (4 points)

For what values of \(x, y, z\) is the code fragment

\[
\texttt{glBegin(GL_POINTS);}
\texttt{glVertex3f(x, y, z);} \\
\texttt{glEnd();}
\]

equivalent to this fragment?

\[
\texttt{glPushMatrix();}
\texttt{glRotatef(180, 0, 1, 0);} \\
\texttt{glScalef(1, 2, 1);} \\
\texttt{glTranslatef(0, 1, 1);} \\
\texttt{glBegin(GL_POINTS);} \\
\texttt{glVertex3f(1, 2, 3);} \\
\texttt{glEnd();} \\
\texttt{glPopMatrix();}
\]

Recall that the operations are specified to OpenGL in reverse order.

\[
\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \text{ translate } \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix} \text{ scale } \begin{bmatrix} 1 \\ 6 \\ 4 \end{bmatrix} \text{ rotate } \begin{bmatrix} -1 \\ 6 \\ -4 \end{bmatrix}
\]

Thus, \(x = -1, y = 6,\) and \(z = -4.\)
Problem 2 (4 points)

Construct an extension of the midpoint algorithm that rasterizes the boundary of the object described by the equation $x^4 + y^4 = r^4$, sketched at right.

- Your solution should be similar to C++ in syntax.
- The signature of the function you are implementing is `void Rasterize_Shape(int r);`
- You may call `void Set_Pixel(int x, int y);` to turn on pixels. It is okay to call it with positive or negative values.
- Your code does not need to be incremental, but it should not use floating point (directly or indirectly).
- It is sufficient to rasterize the part shown in blue. ($0 \leq x \leq y$).

```c
void Rasterize_Shape(int r)
{
    int x = 0, y = r;
    while (x <= y)
    {
        Set_Pixel(x, y);
        x++;
        if (16*x*x*x*x+(2*y-1)*(2*y-1)*(2*y-1)*(2*y-1)>16*r*r*r*r)
            y--;
    }
}
```
Problem 3 (4 points)
The triangle at right is to be rasterized. The colors of the vertices are \( A = \text{red} = (1, 0, 0), \) \( B = \text{green} = (0, 1, 0) \) and, \( C = \text{blue} = (0, 0, 1) \). Compute the color of the point \( P \).

To do this, we need to compute the areas of the triangles, which we can then use to compute barycentric weights.

\[
\begin{align*}
\text{area}(ABC) &= 18 \\
\text{area}(PBC) &= 8 \\
\text{area}(APC) &= 6 \\
\text{area}(ABP) &= 4
\end{align*}
\]

\[
\begin{align*}
\alpha &= \frac{\text{area}(PBC)}{\text{area}(ABC)} = \frac{4}{9} \\
\beta &= \frac{\text{area}(APC)}{\text{area}(ABC)} = \frac{3}{9} \\
\gamma &= \frac{\text{area}(ABP)}{\text{area}(ABC)} = \frac{2}{9}
\end{align*}
\]

\[
C_P = \alpha C_A + \beta C_B + \gamma C_C = \left( \frac{4}{9}, \frac{3}{9}, \frac{2}{9} \right)
\]
Problem 4 (4 points)

What initial values of $x, y, z$ in `barB` will make the routines `barA` and `barB` equivalent?

```c
void barA(float a, float b, float c, int n)
{
    for (int i = 0; i < n; i++)
    {
        float z = a * i * i + b * i + c;
        foo(z);
    }
}

void barB(float a, float b, float c, int n)
{
    float x = ???;
    float y = ???;
    float z = ???;
    for (int i = 0; i < n; i++)
    {
        foo(z);
        z += y;
        y += x;
    }
}
```

Note that in `barA`, the first three calls to `foo` are `foo(c)`, `foo(a+b+c)`, and `foo(4*a+2*b+c)`. In `barB`, the first three calls to `foo` are `foo(z)`, `foo(z+y)`, and `foo(z+2*y+x)`. Since the first calls are the same, $z=c$. Since the second calls are the same, $y=a+b$. To make the third calls the same, we need $x=2*a$. This is the simplest way to find the initial values.

```c
void barB(float a, float b, float c, int n)
{
    float x = 2 * a;
    float y = a + b;
    float z = c;
    for (int i = 0; i < n; i++)
    {
        foo(z);
        z += y;
        y += x;
    }
}
```
Problem 5 (4 points)
Show step by step how OpenGL would clip the triangle below against the square. (Draw a new diagram each time you clip a triangle.)

Here is one way it may occur.
Problem 6 (1 point each part)

Let \( f(t) = (4t, -t + 1, t - 1, t + 1) \) define a curve in homogeneous coordinates for \( t \in [0, 1] \).

(a) Express \( f(t) \) in non-homogeneous 3D coordinates.

(b) Using (a), show that this corresponds to a line. (This is not at all obvious from the homogeneous representation.)

(a)

\[
f(t) = \begin{pmatrix} 4t \\ -t + 1 \\ t - 1 \\ t + 1 \end{pmatrix}
\]

(b) One way to do this is to show that it can be expressed in the form

\[
f(t) = \begin{pmatrix} 4t \\ -t + 1 \\ t + 1 \end{pmatrix} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + s \begin{pmatrix} d \\ e \\ f \end{pmatrix} = g(s)
\]

\[
f(0) = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}
\]

\[
f(t) = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + \begin{pmatrix} \frac{4t}{t+1} \\ \frac{2t}{t+1} - 1 \\ \frac{2t}{t+1} \end{pmatrix}
\]

\[
= \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + 2t \begin{pmatrix} \frac{2}{t+1} \\ -1 \\ 1 \end{pmatrix}
\]

\[
= \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + s \begin{pmatrix} \frac{2}{t+1} \\ -1 \\ 1 \end{pmatrix}
\]
Problem 7 (1 point each part)

Let \( f(t) = (4t, -t + 1, t - 1, t + 1) \) define a curve in homogeneous coordinates for \( t \in [0, 1] \).

(a) What are the values of \( f(t) \) at its endpoints (express your answer in non-homogeneous 3D coordinates)?

(b) Construct (in non-homogeneous 3D coordinates) an equation \( g(s) = \vec{u} + \vec{v}s \) for this line by using the endpoints you found in (a). You may assume that the curve is in fact a line, even if you did not show this.

(a)

\[
f(t) = \begin{pmatrix} 4t \\ -t + 1 \\ t - 1 \\ t + 1 \end{pmatrix}
\]

\[
f(0) = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}
\]

\[
f(1) = \begin{pmatrix} 4 \\ 0 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}
\]

(b)

\[
g(s) = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + s \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}
\]

\[
= \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} + s \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}
\]
Problem 8 (1 point each part)
The clipping region in homogeneous equations (3D) is given by $-w \leq x \leq w$, $-w \leq y \leq w$, and $-w \leq z \leq w$. Let $f(t) = (4t, -t+1, t-1, t+1)$ define a curve in homogeneous coordinates for $t \in [0, 1]$.
(a) Clip the equation for the line $f(t)$ in homogeneous coordinates against the clipping constraint $-w \leq x \leq w$. (Find the value of $t$ and homogeneous coordinates for the clipping point.)

(b) What is the clipping constraint corresponding to clip the equation $-w \leq x \leq w$ in non-homogeneous coordinates?

(a)

\[-w = x\]
\[-(t+1) = 4t\]
\[-1 = 5t\]
\[t = -\frac{1}{5}\]

This is not an intersection, since $t \not\in [0, 1]$.

\[w = x\]
\[t + 1 = 4t\]
\[1 = 3t\]
\[t = \frac{1}{3}\]

\[f\left(\frac{1}{3}\right) = \begin{pmatrix} 4 \\ -2/3 \\ -2/3 \\ 4/3 \end{pmatrix}\]

(b)

\[-w \leq x \leq w\]
\[-1 \leq \frac{x}{w} \leq 1\]
\[-1 \leq x' \leq 1\]