Background

Review material

- **Sets and relations:** finite set, infinite set, countable set, uncountable set, cross-product, reflexive relation, transitive relation, symmetric relation, antisymmetric relation, equivalence relation, partial order relation, total order relation
- **Graphs:** directed graph, undirected graph, path, cycle, connected graph, strongly connected component (SCC), clique, distance, depth-first search (DFS), breadth-first search (BFS), tree, binary search tree, directed acyclic graph (DAG), bipartite graph
- **Formal languages:** regular expression, deterministic finite automaton, nondeterministic finite automaton, accepting state, language accepted by an automaton, regular grammar, context-free grammar, pushdown automaton, Turing machine, unrestricted grammar
- **Computability:** Turing-decicdable language, Turing-acceptable language, the halting problem
- **Asymptotic notation:** $O(f(n))$, $\Omega(f(n))$, $\Theta(f(n))$
- **Complexity:** increasingly complex classes: *log*, *polylog*, *linear*, *polynomial*, *exponential*, *doubly exponential*, *nonelementary*; deterministic vs. nondeterministic, time vs. space complexity: NL(SPACE), P(TIME), NP(TIME), PSPACE, EXPTIME, EXPSPACE, NONELEMENTARY
- **Linear algebra:** vector, matrix, inner product of vectors, identity matrix, transpose, linear independent set of vectors, rank of a matrix
- **Important sets:** $\mathbb{B} = \{0, 1\}$, the booleans; $\mathbb{N} = \{0, 1, 2, ...\}$, the natural numbers; $\mathbb{Z} = \{..., -2, -1, 0, 1, 2, ...\}$, the integers; $\mathbb{Q}$, the rational numbers; $\mathbb{R}$, the real numbers

First order logic

A *signature* $G = (\mathcal{V}, \mathcal{F}, \mathcal{R})$ specifies

- $\mathcal{V}$: the variable symbols, taking value over a predefined domain $\mathcal{X}$ (e.g., the integers)
- $\mathcal{F}$: the function symbols, $f : \mathcal{X}^n \rightarrow \mathcal{X}$
- $\mathcal{R}$: the relation symbols, $r : \mathcal{X}^n \rightarrow \{False, True\}$

Each function or relation has an *arity* n

- the arity of function sum is 2 (e.g., talking about integer sum)
- the arity of relation $less$ is 2 (e.g., talking about integer comparison)
- a constant is a function of arity 0

The syntax for a formula in first order logic is:

$$
\begin{array}{lcl}
form & ::= & simp \mid (form \wedge form) \mid (form \vee form) \mid (form \Rightarrow form) \mid \\
 & & (\neg form) \mid \forall var(form) \mid \exists var(form) \mid True \mid False \\
simp & ::= & rel(term, term, ..., term) \mid term \equiv term \\
term & ::= & var \mid const \mid func(term, term, ..., term)
\end{array}
$$

Propositional logic

Propositional logic formulas admit

- no quantification (no $\forall$, no $\exists$)
- no function symbols
- no relation symbols

Thus the signature is $(\mathcal{A}, \emptyset, \emptyset)$

Each propositional variable in the set $\mathcal{A}$ taks value in $\{True, False\}$ (or $\mathbb{B}$)

The syntax of a propositional logic formula is:

$$f ::= p \mid (f \wedge f) \mid (f \vee f) \mid (f \Rightarrow f) \mid \neg f \mid True \mid False$$

where $p \in \mathcal{A}$

An assignment a gives a value to each propositional variable $a : \mathcal{A} \rightarrow \{True, False\}$