



FIGURE 2 The Graph of $f(x) = \log x$.

By part 2 of Theorem 1, we have

$$\begin{aligned} b^{\log_a x \cdot \log_b a} &= (b^{\log_b a})^{\log_a x} \\ &= a^{\log_a x} \\ &= x. \end{aligned}$$

This completes the proof. ◀

Because the base used most often for logarithms in this text is $b = 2$, the notation $\log x$ is used throughout the text to denote $\log_2 x$.

The graph of the function $f(x) = \log x$ is displayed in Figure 2. From Theorem 3, when a base b other than 2 is used, a function that is a constant multiple of the function $\log x$, namely, $(1/\log b) \log x$, is obtained.

Exercises

1. Express each of the following quantities as powers of 2.
 - a) $2 \cdot 2^2$
 - b) $(2^2)^3$
 - c) $2^{(2^2)}$
2. Find each of the following quantities.
 - a) $\log_2 1024$
 - b) $\log_2 1/4$
 - c) $\log_4 8$
3. Suppose that $\log_4 x = y$ where x is a positive real number. Find each of the following quantities.
 - a) $\log_2 x$
 - b) $\log_8 x$
 - c) $\log_{16} x$
4. Let a , b , and c be positive real numbers. Show that $a^{\log_b c} = c^{\log_b a}$. ✎
5. Draw the graph of $f(x) = b^x$ for all real numbers x if b is
 - a) 3.
 - b) $1/3$.
 - c) 1.
6. Draw the graph of $f(x) = \log_b x$ for positive real numbers x if b is
 - a) 4.
 - b) 100.
 - c) 1000.