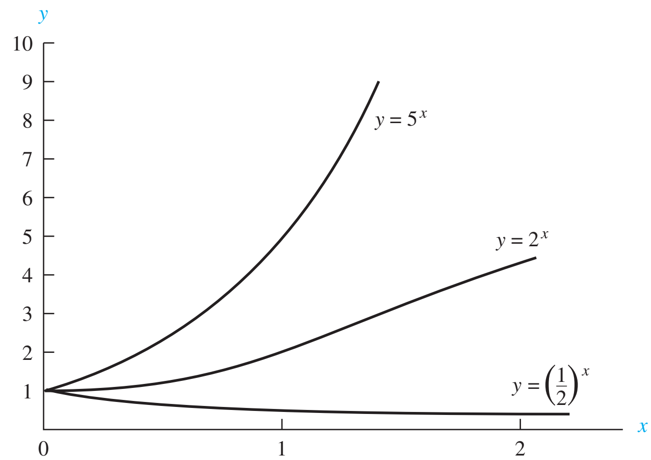

FIGURE 1 **Graphs of the Exponential Functions to the Bases $\frac{1}{2}$, 2, and 5.**

THEOREM 2 Let b be a real number greater than 1. Then

1. $\log_b(xy) = \log_b x + \log_b y$ whenever x and y are positive real numbers, and
2. $\log_b(x^y) = y \log_b x$ whenever x is a positive real number and y is a real number.

Proof: Because $\log_b(xy)$ is the unique real number with $b^{\log_b(xy)} = xy$, to prove part 1 it suffices to show that $b^{\log_b x + \log_b y} = xy$. By part 1 of Theorem 1, we have

$$\begin{aligned} b^{\log_b x + \log_b y} &= b^{\log_b x} b^{\log_b y} \\ &= xy. \end{aligned}$$

To prove part 2, it suffices to show that $b^{y \log_b x} = x^y$. By part 2 of Theorem 1, we have

$$\begin{aligned} b^{y \log_b x} &= (b^{\log_b x})^y \\ &= x^y. \end{aligned}$$

◁

The following theorem relates logarithms to two different bases.

THEOREM 3 Let a and b be real numbers greater than 1, and let x be a positive real number. Then

$\log_a x = \log_b x / \log_b a$.

Proof: To prove this result, it suffices to show that

$$b^{\log_a x \cdot \log_b a} = x.$$