



**FIGURE 2** The Graph of  $f(x) = \log x$ .

By part 2 of Theorem 1, we have

$$\begin{aligned} b^{\log_a x \cdot \log_b a} &= (b^{\log_b a})^{\log_a x} \\ &= a^{\log_a x} \\ &= x. \end{aligned}$$

This completes the proof. ◀

Because the base used most often for logarithms in this text is  $b = 2$ , the notation  $\log x$  is used throughout the text to denote  $\log_2 x$ .

The graph of the function  $f(x) = \log x$  is displayed in Figure 2. From Theorem 3, when a base  $b$  other than 2 is used, a function that is a constant multiple of the function  $\log x$ , namely,  $(1/\log b) \log x$ , is obtained.

## Exercises

1. Express each of the following quantities as powers of 2.
  - a)  $2 \cdot 2^2$
  - b)  $(2^2)^3$
  - c)  $2^{(2^2)}$
2. Find each of the following quantities.
  - a)  $\log_2 1024$
  - b)  $\log_2 1/4$
  - c)  $\log_4 8$
3. Suppose that  $\log_4 x = y$  where  $x$  is a positive real number. Find each of the following quantities.
  - a)  $\log_2 x$
  - b)  $\log_8 x$
  - c)  $\log_{16} x$
4. Let  $a$ ,  $b$ , and  $c$  be positive real numbers. Show that  $a^{\log_b c} = c^{\log_b a}$ . ▶
5. Draw the graph of  $f(x) = b^x$  for all real numbers  $x$  if  $b$  is
  - a) 3.
  - b)  $1/3$ .
  - c) 1.
6. Draw the graph of  $f(x) = \log_b x$  for positive real numbers  $x$  if  $b$  is
  - a) 4.
  - b) 100.
  - c) 1000.