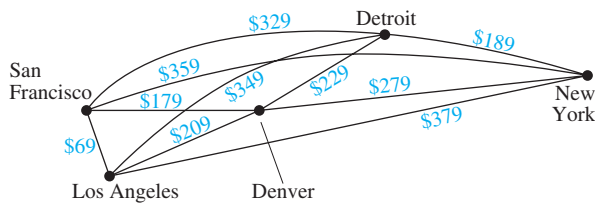


27. Find a route with the least total airfare that visits each of the cities in this graph, where the weight on an edge is the least price available for a flight between the two cities.



28. Find a route with the least total airfare that visits each of the cities in this graph, where the weight on an edge is the least price available for a flight between the two cities.



29. Construct a weighted undirected graph such that the total weight of a circuit that visits every vertex at least once is minimized for a circuit that visits some vertices more than once. [Hint: There are examples with three vertices.]

30. Show that the problem of finding a circuit of minimum total weight that visits every vertex of a weighted graph at least once can be reduced to the problem of finding a circuit of minimum total weight that visits each vertex of a weighted graph exactly once. Do so by constructing a new weighted graph with the same vertices and edges as the original graph but whose weight of the edge connecting the vertices u and v is equal to the minimum total weight of a path from u to v in the original graph.

- *31. The **longest path problem** in a weighted directed graph with no simple circuits asks for a path in this graph such that the sum of its edge weights is a maximum. Devise an algorithm for solving the longest path problem. [Hint: First find a topological ordering of the vertices of the graph.]

10.7 Planar Graphs

Introduction



Consider the problem of joining three houses to each of three separate utilities, as shown in Figure 1. Is it possible to join these houses and utilities so that none of the connections cross? This problem can be modeled using the complete bipartite graph $K_{3,3}$. The original question can be rephrased as: Can $K_{3,3}$ be drawn in the plane so that no two of its edges cross?

In this section we will study the question of whether a graph can be drawn in the plane without edges crossing. In particular, we will answer the houses-and-utilities problem.

There are always many ways to represent a graph. When is it possible to find at least one way to represent this graph in a plane without any edges crossing?

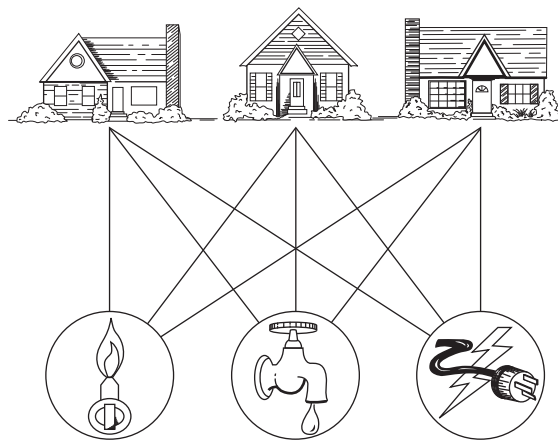


FIGURE 1 Three Houses and Three Utilities.