UCR CS 141 Spring 2003
Midterm exam C

Name: ______________________ S_ID: _________________________

1. Illustrate the performance of the merge-sort algorithm on the following input sequence: (22, 15, 36, 44, 10, 3, 9, 13, 29, 25). Show your work by drawing the tree at each step. (15 pts)

2. A complex number $a + b\mathbf{i}$ where $\mathbf{i} = \sqrt{-1}$, can be represented by the pair (a, b). Describe a method performing only **three** real-number multiplications to compute the pair (e, f) representing the product of $a + b\mathbf{i}$ and $c + d\mathbf{i}$. (**Hint:** *Similar to integer multiplication covered in class*) (20 pts)

3. Describe, in pseudo code, a link-hopping method for finding the middle node of a doubly linked list with header and trailer sentinels, and an **odd** number of real nodes between them. (**Hints:** *Do not use a counter, just link hopping. Follow links simultaneously from head and tail an make comparison at each step*) (20 pts)

4. Describe, using pseudo code, a method for multiplying an n x m matrix A and an m x p matrix B. Recall that the product C = AB is defined so that C[i][j] = $\sum_{k=1}^{m}$ A[i][k] . B[k][j]. (*Your method may not be the most efficient, but it should work!*) (15 pts)

5. Characterize each of the following recurrence equations using the master method (assuming that $T(n) = c$ for $n < d$, for constants $c > 0$ and $d \geq 1$). (30 pts, 10 pts each)
   a. $T(n) = 2T(n / 2) + \log n$
   b. $T(n) = 8T(n / 2) + n^2$
   c. $T(n) = 16T(n / 2) + (n \log n)^4$

   Hint: Rules for Master Theorem

   1. if $f(n)$ is $O(n^{\log_b a-\varepsilon})$, then $T(n)$ is $\Theta(n^{\log_b a})$
   2. if $f(n)$ is $\Theta(n^{\log_b a} \log^k n)$, then $T(n)$ is $\Theta(n^{\log_b a} \log^{k+1} n)$
   3. if $f(n)$ is $\Omega(n^{\log_b a+\varepsilon})$, then $T(n)$ is $\Theta(f(n))$,
      provided $af(n/b) \leq \delta f(n)$ for some $\delta < 1$.

Good Luck!