

CS 140A - Homework 1
Due Monday, October 13

1. (a) Is $2^{n+1} = O(2^n)$?
(b) Is $2^{2n} = O(2^n)$?
2. Order the following functions by growth rate:
 $N, \sqrt{N}, N^{1.5}, N \log N, N^2, N \log^2 N, N \log(N^2), 2^N, 37, N^2 \log N, N^3$. Indicate which functions grow at the same rate.
3. For each of the following program fragments:
 - (a) Write a function to describe the running time of the code. Assume that a single statement takes one unit of running time.
 - (b) Give an analysis of the running time in Big-Oh notation.
 - (c) Implement the code, and give the running time for several values of N . You can measure the actual running time using the “time” command under Unix.
 - (d) Compare your analysis with the actual running times.
 - i.

```
sum = 0;
for(i = 0; i < N; i++)
    sum++;
```
 - ii.

```
sum = 0;
for(i = 0; i < N; i++)
    for(j = 0; j < N; j++)
        sum++;
```
 - iii.

```
sum = 0;
for(i = 0; i < N; i++)
    for(j = 0; j < N * N; j++)
        sum++;
```
 - iv.

```
sum = 0;
for(i = 0; i < N; i++)
    for(j = 0; j < i; j++)
        sum++;
```
 - v.

```
sum = 0;
for(i = 0; i < N; i++)
    for(j = 0; j < i * i; j++)
        sum++;
```
4. Programs A and B are analyzed and found to have worst-case running times no greater than $150N \log_2 N$ and N^2 , respectively. Answer the following questions.

- (a) Which program has the better guarantee on the running time, for large values of N ($N > 10,000$)?
- (b) Which program has the better guarantee on the running time, for small values of N ($N < 100$)?
- (c) Which program will run faster on average for $N = 1,000$?
- (d) Is it possible that program B will run faster than program A on all possible inputs?