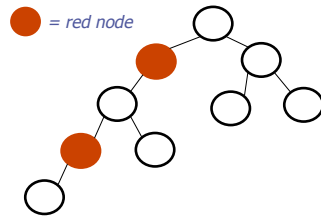


Red-Black Trees



Red-Black Trees

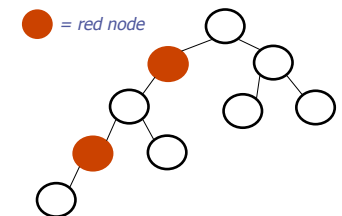
- ◆ Each node contains an extra bit to keep track of its color
 - Each node is either **red** or **black**
- ◆ The coloring ensures that no path on the way from the root to a leaf is more than twice as long as any other path
 - APPROXIMATELY BALANCED - close enough

Red-Black Tree Properties

- ◆ A red-black tree is a BST with the following coloring properties
 1. Every node is colored either red or black
 2. The root is black
 3. If a node is red, its children must be black
 4. Every path from a node to a NULL pointer must contain the same number of black nodes

Red-Black Trees

- ◆ NULL pointers are considered black nodes
- ◆ Black-height = the number of black nodes on any path from, but not including, a node to a leaf



Balance

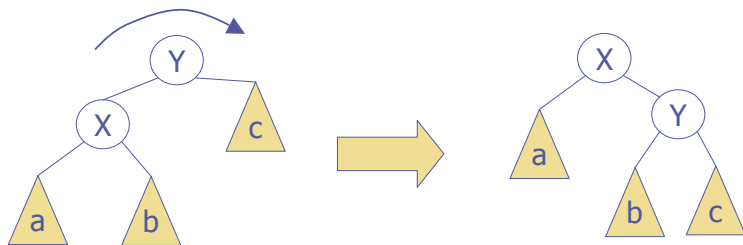
- ◆ Due to the coloring (balancing), the height of the tree is at most $2\lg(N+1)$
- ◆ Search is the same as for a BST - $O(\lg n)$
- ◆ When inserting and removing, the tree must be adjusted so the properties of the red-black tree still hold

Re-Balancing

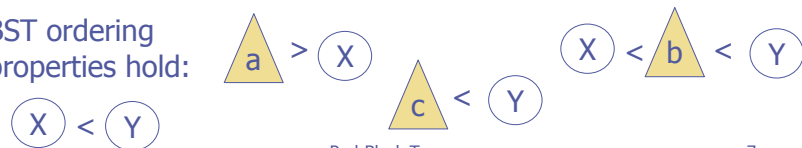
- ◆ When inserting and removing we rebalance the tree by doing rotations and recoloring

Right Rotate

Right rotate around y

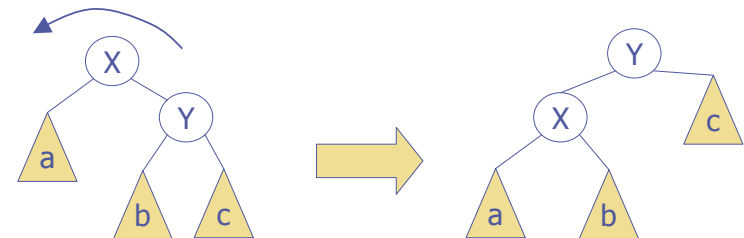


BST ordering properties hold:

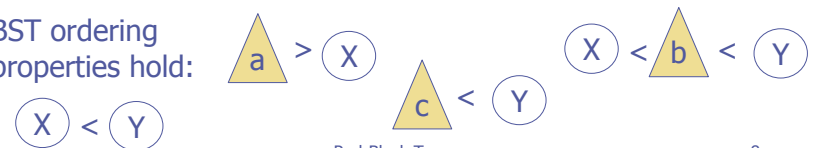


Left Rotate

Left rotate around X

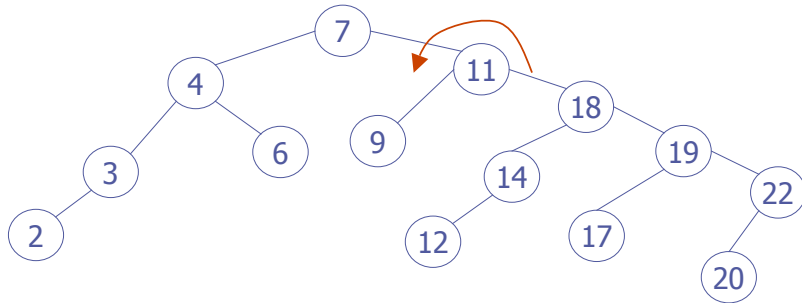


BST ordering properties hold:



Left Rotate

◆ In class exercise - left rotate around 11



Inserting Into a Red-Black Tree

- ◆ Insert a node using the normal BST insert method
- ◆ Color the node red
- ◆ Use rotations and recolor nodes to maintain red-black tree properties

Inserting Into a Red-Black Tree

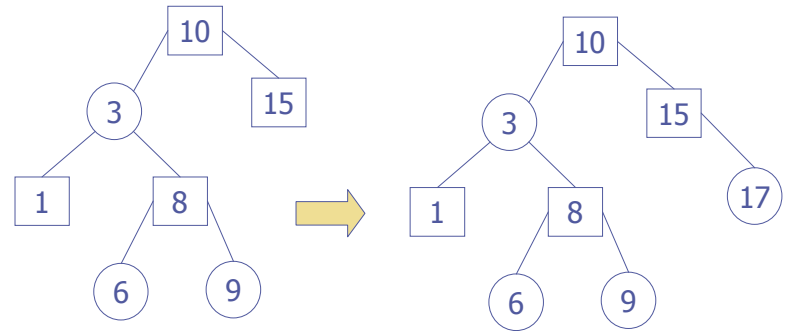
- ◆ Possible actions
 1. If the parent of the newly inserted node is black, color the node red and we are done
 2. If the parent of the newly inserted node is red - violation of property 3, two possible cases
 1. Sibling of parent (uncle) is black - rotate depending on whether or not x is a right child
 2. Sibling of parent (uncle) is red - recolor

Inserting Into a Red-Black Tree

◆ Fixing the local problem may create another problem so percolate up

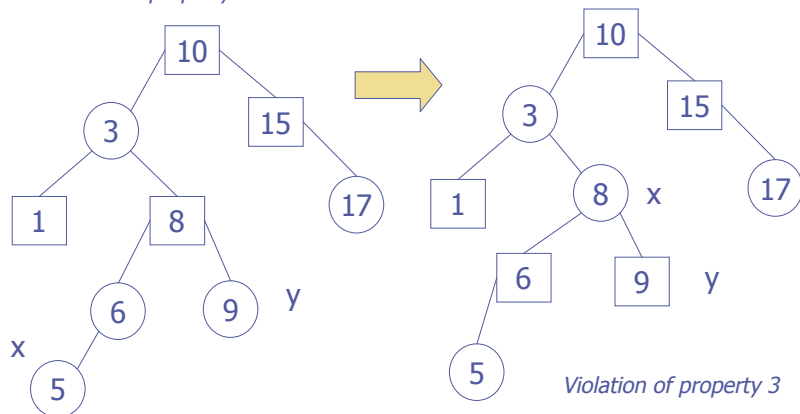
Insert 17

10 = black node

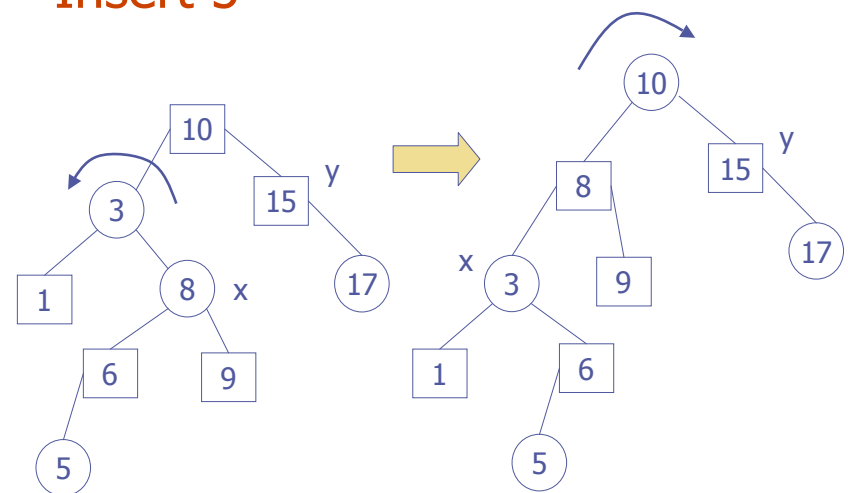


Insert 5

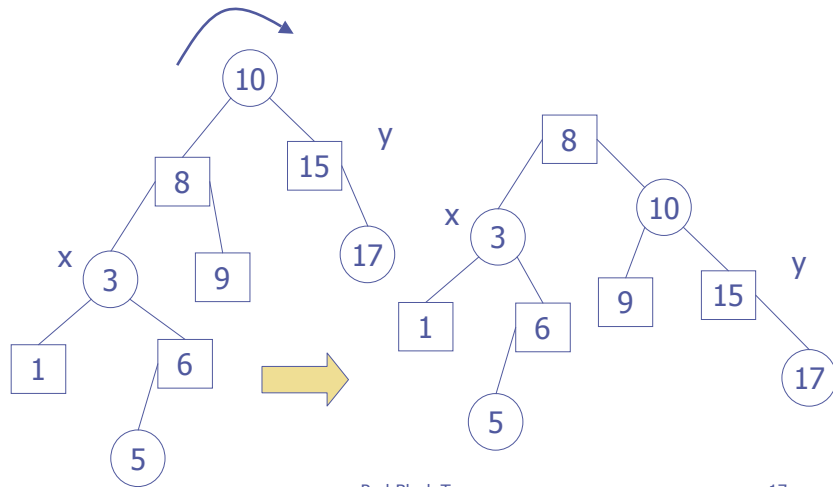
Violation of property 3



Insert 5



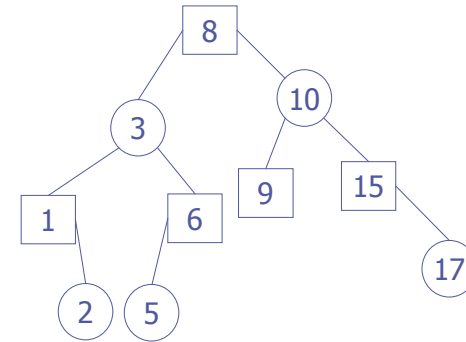
Insert 5



Red-Black Trees

17

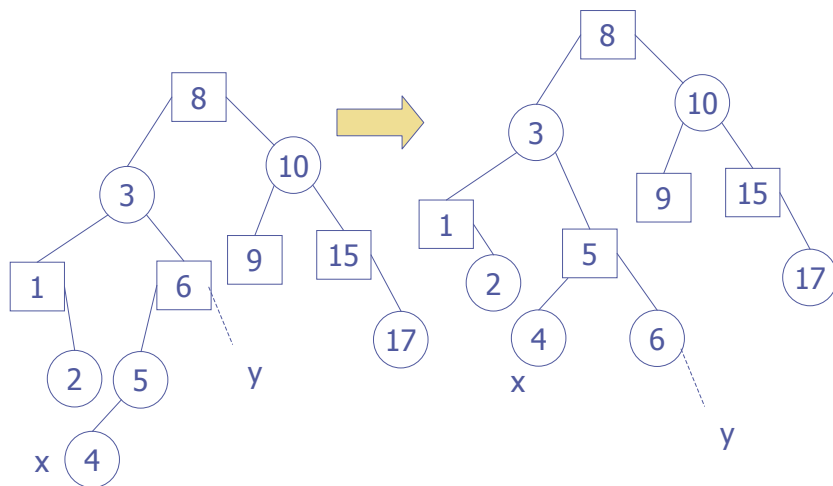
Insert 2



Red-Black Trees

18

Insert 4



Red-Black Trees

19

Insert 41

- ◆ In class exercise - insert the following values one at a time into an initially empty red-black tree
 - 41, 38, 31, 12, 19, 8

Red-Black Trees

20

Insert 38

Insert 31

Insert 12

Insert 19

Insert 19

Insert 8