First-Come First-Served (FCFS) for Online Slot Allocation (OSA) and Huffman Coding

— SODA 2014 —

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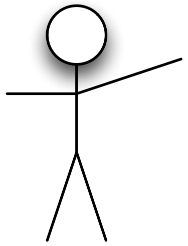
defn of OSA



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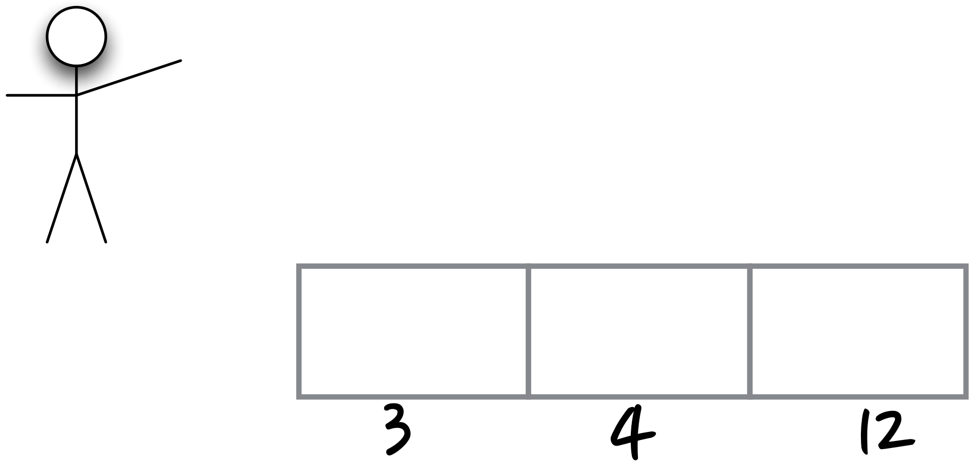

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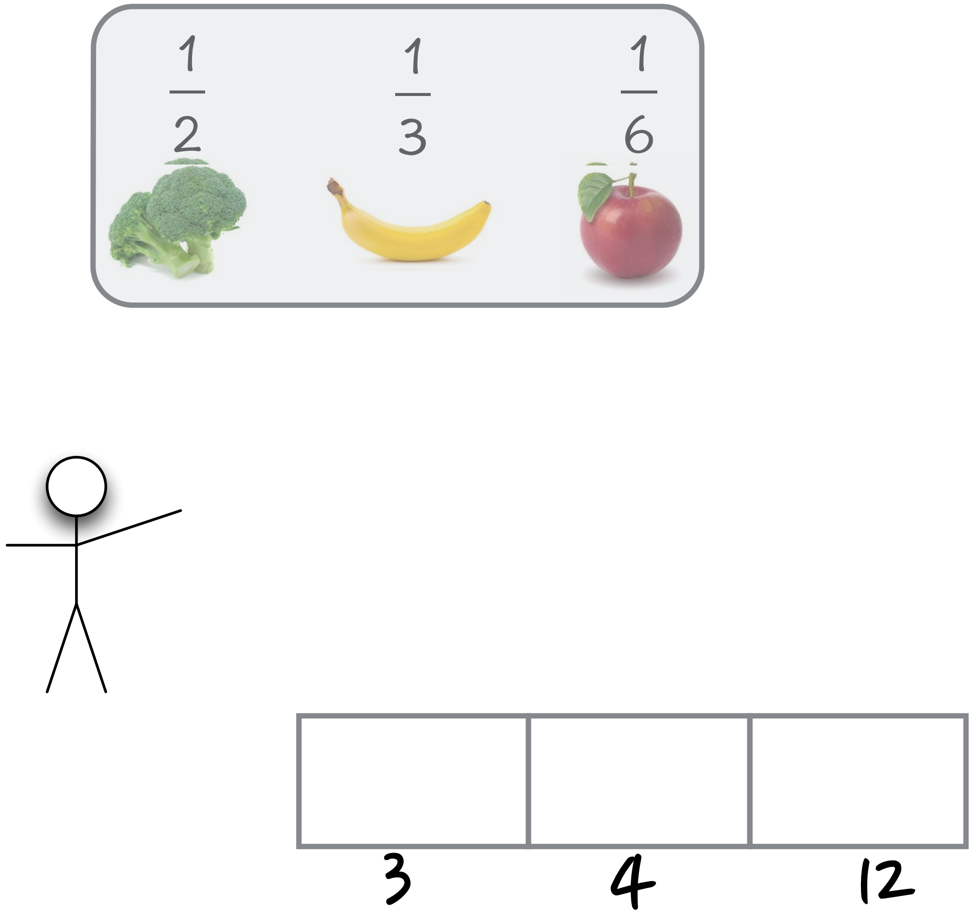

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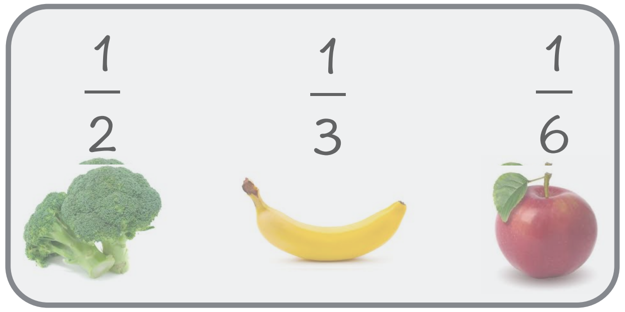

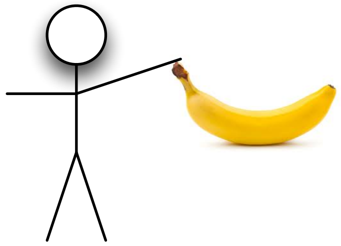

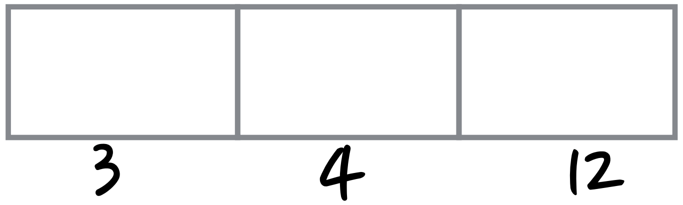

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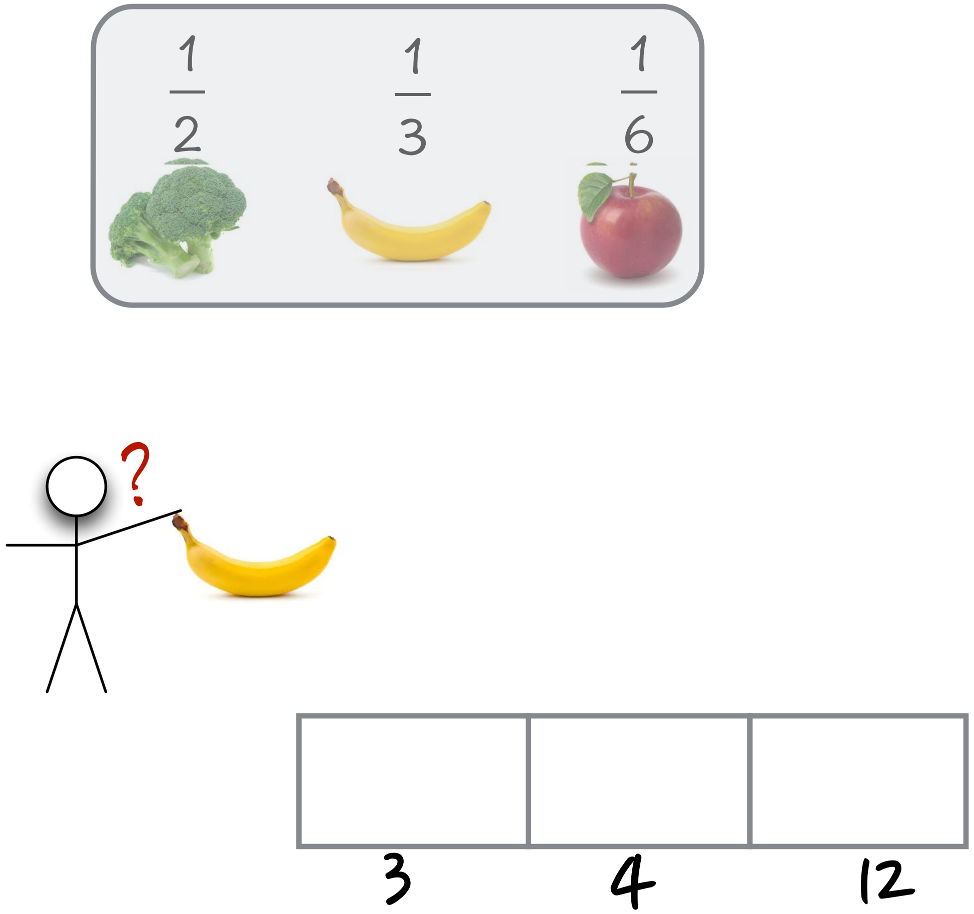

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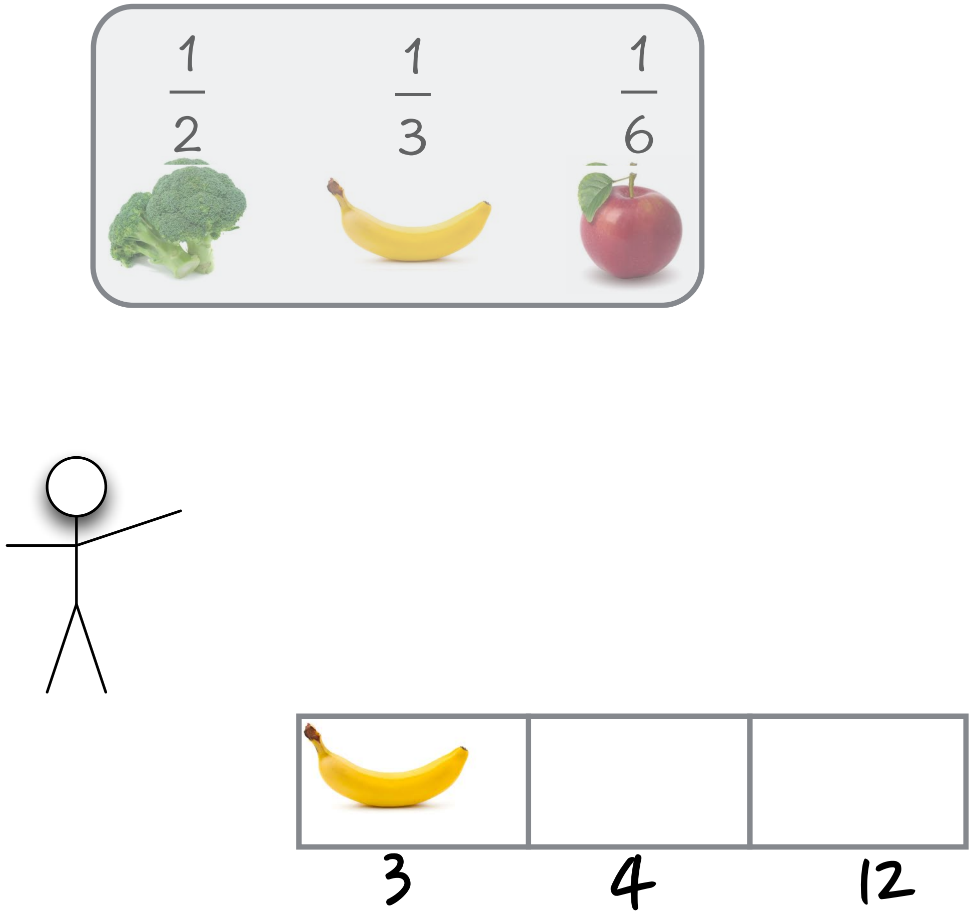

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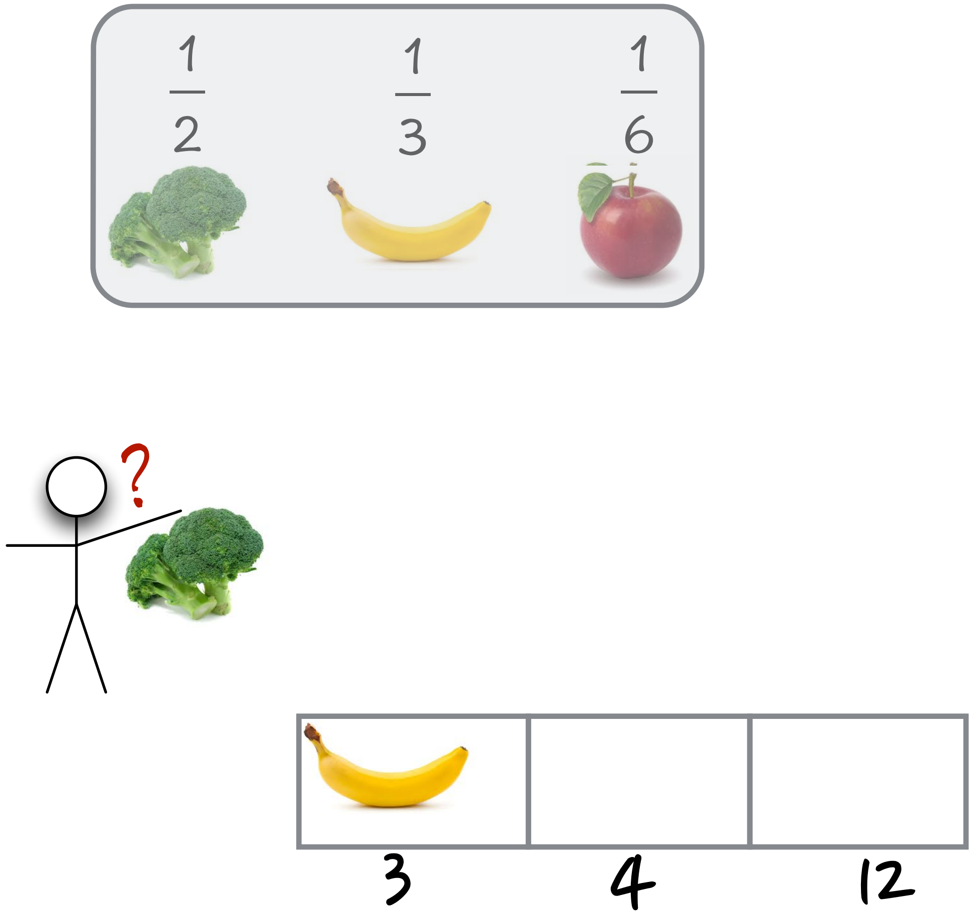

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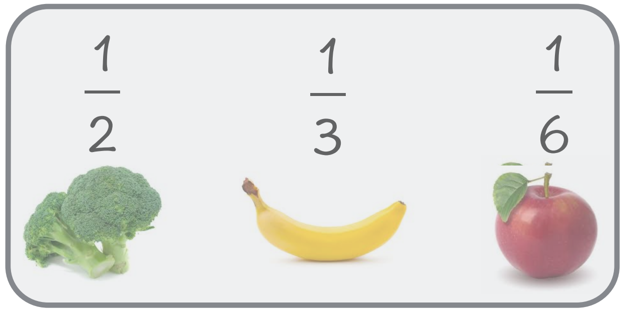

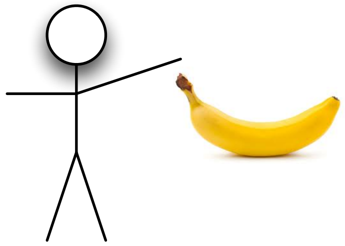

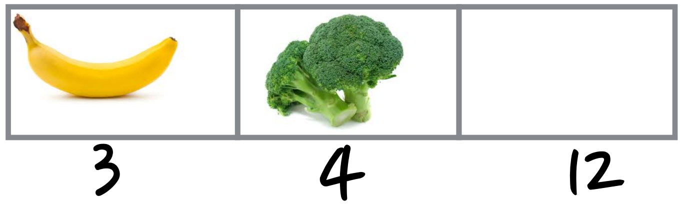

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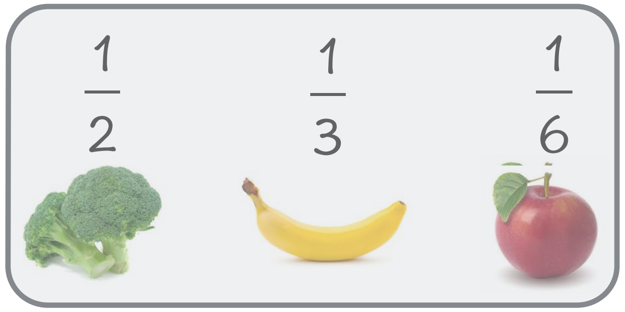

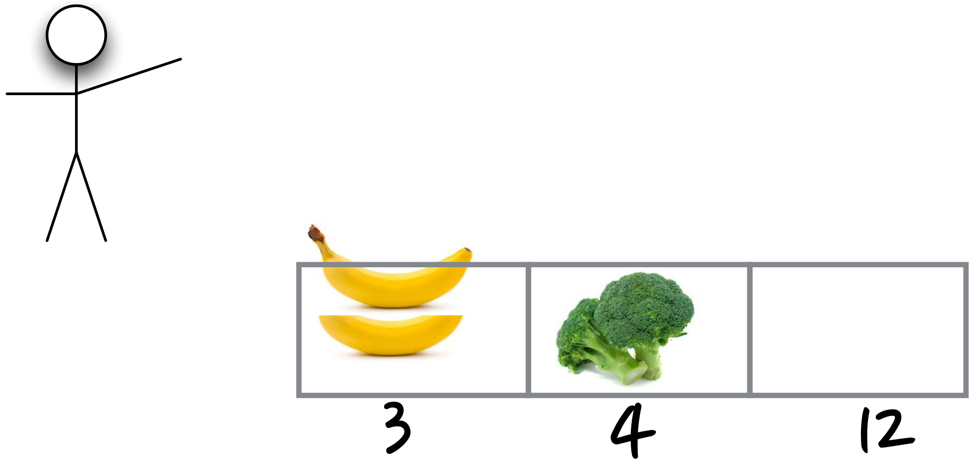

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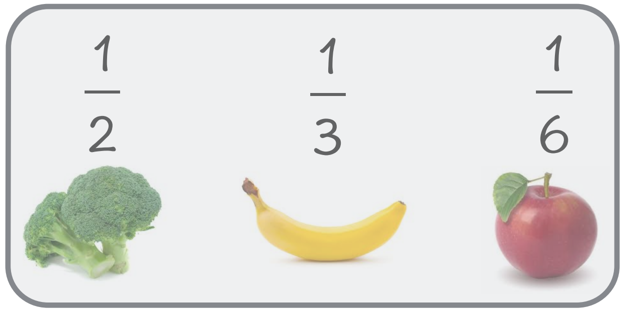

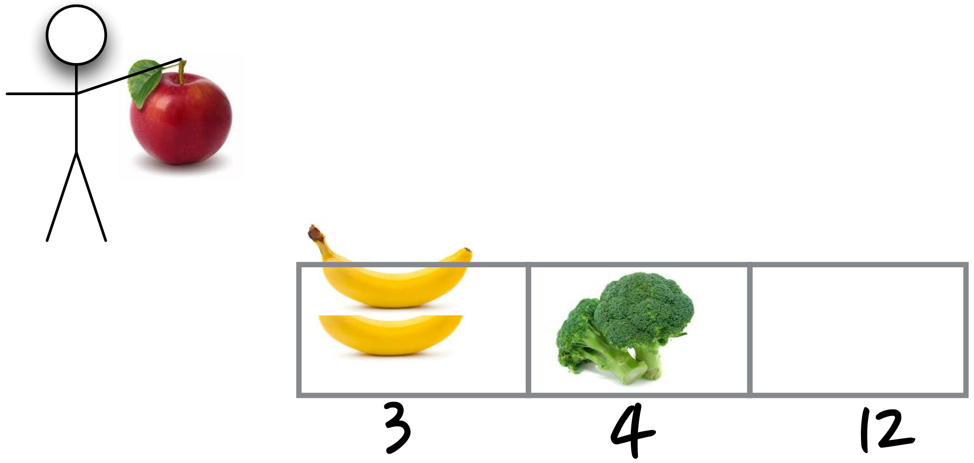

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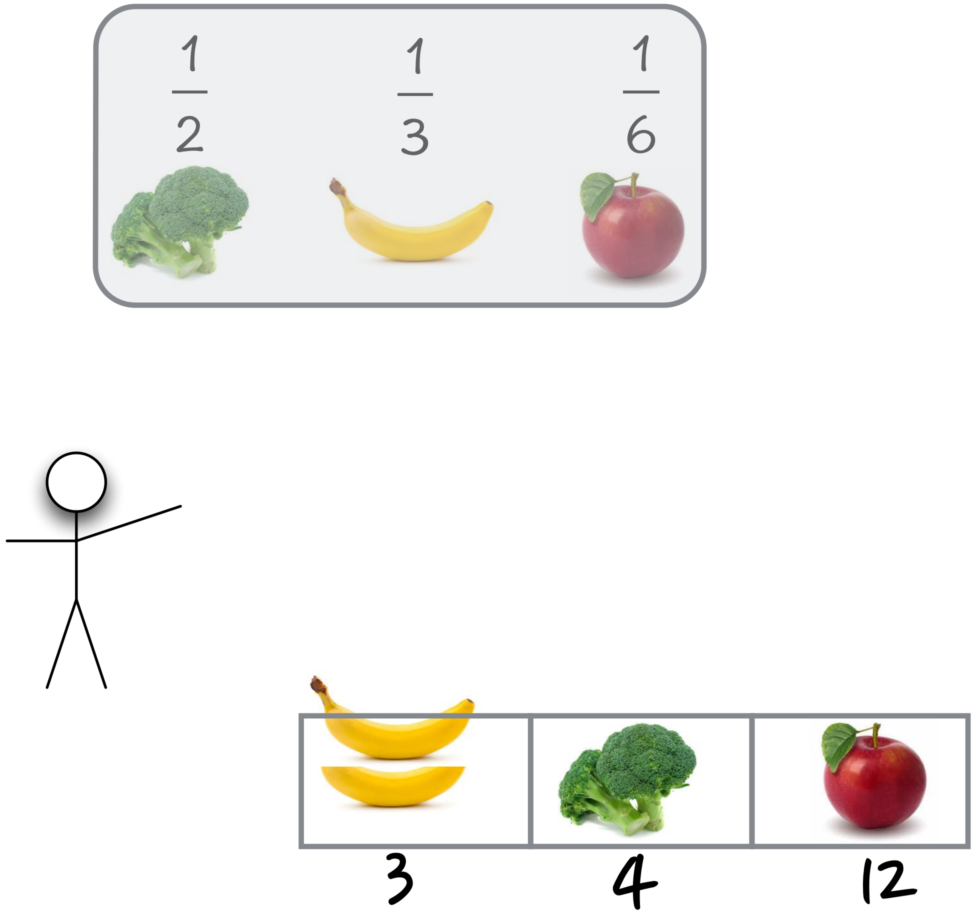

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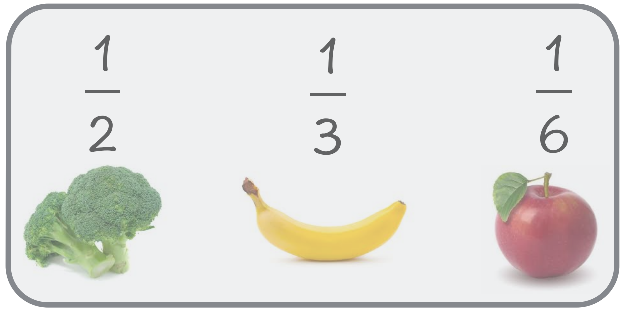

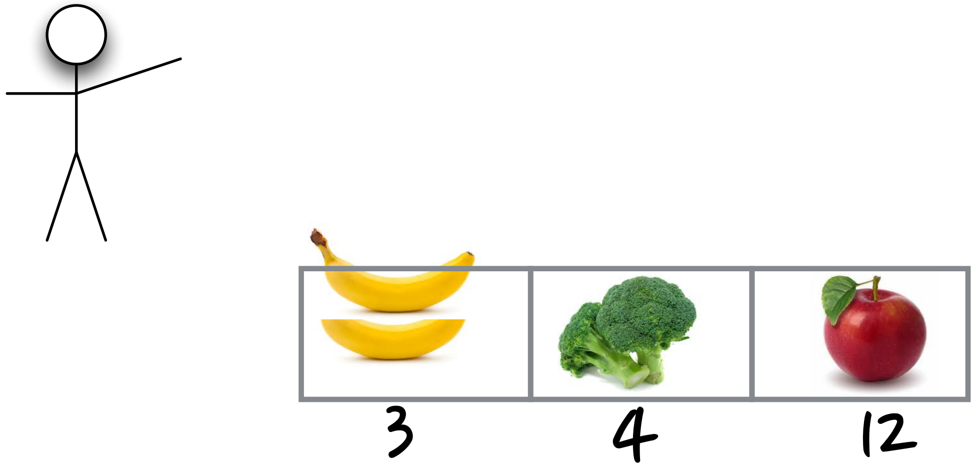

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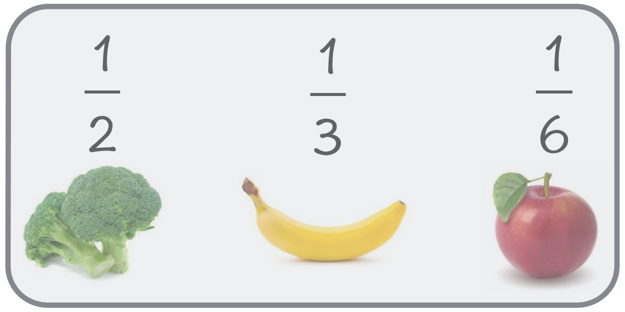

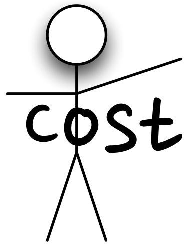

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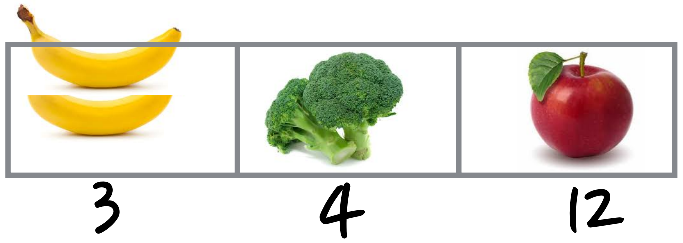

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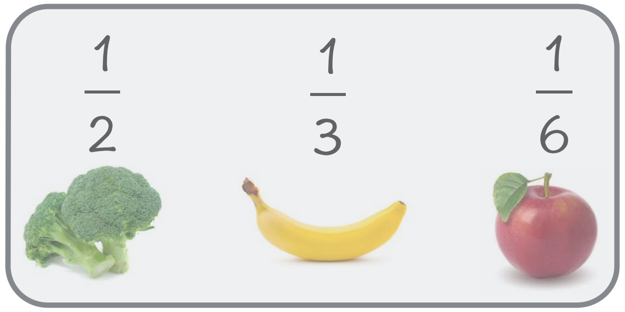

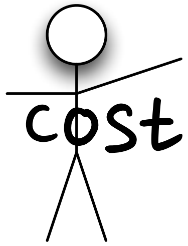

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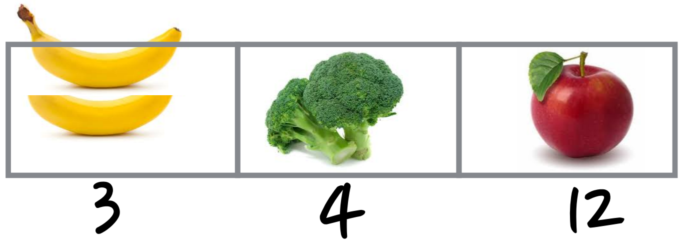

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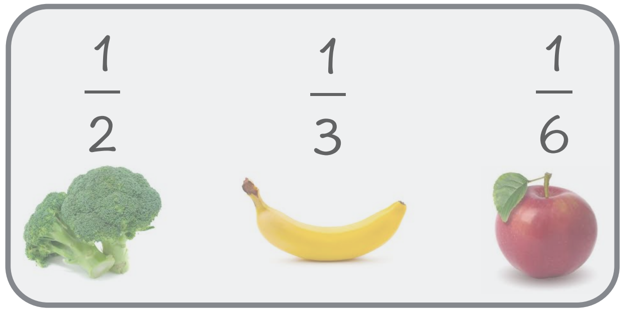

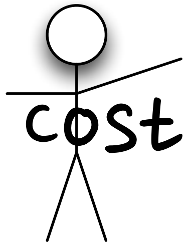

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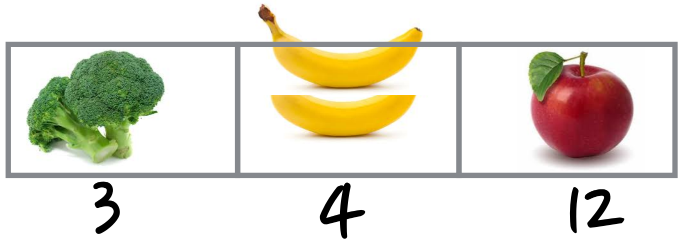

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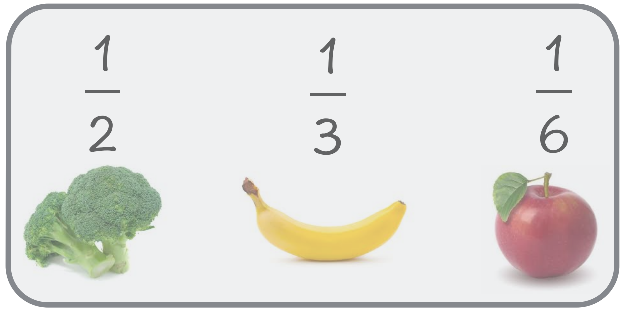

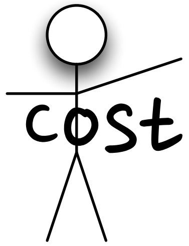

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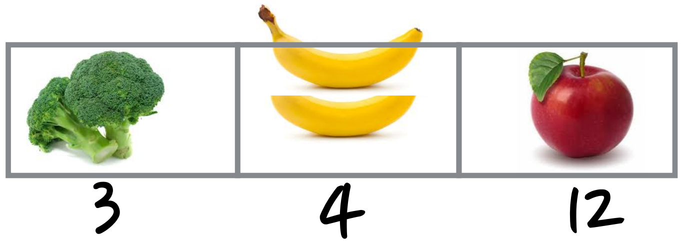

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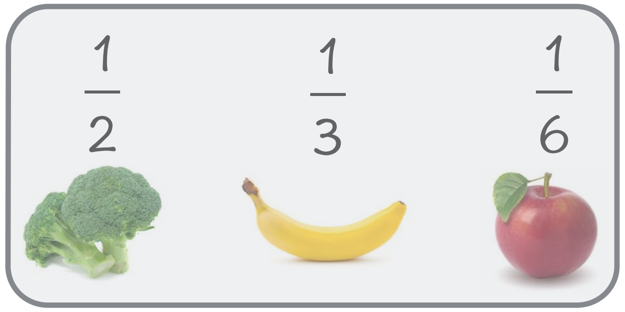
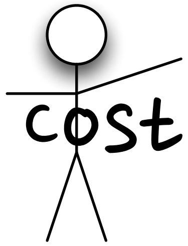


FcFS = use cheapest available slot



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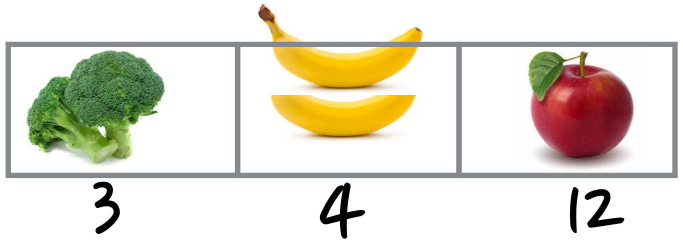

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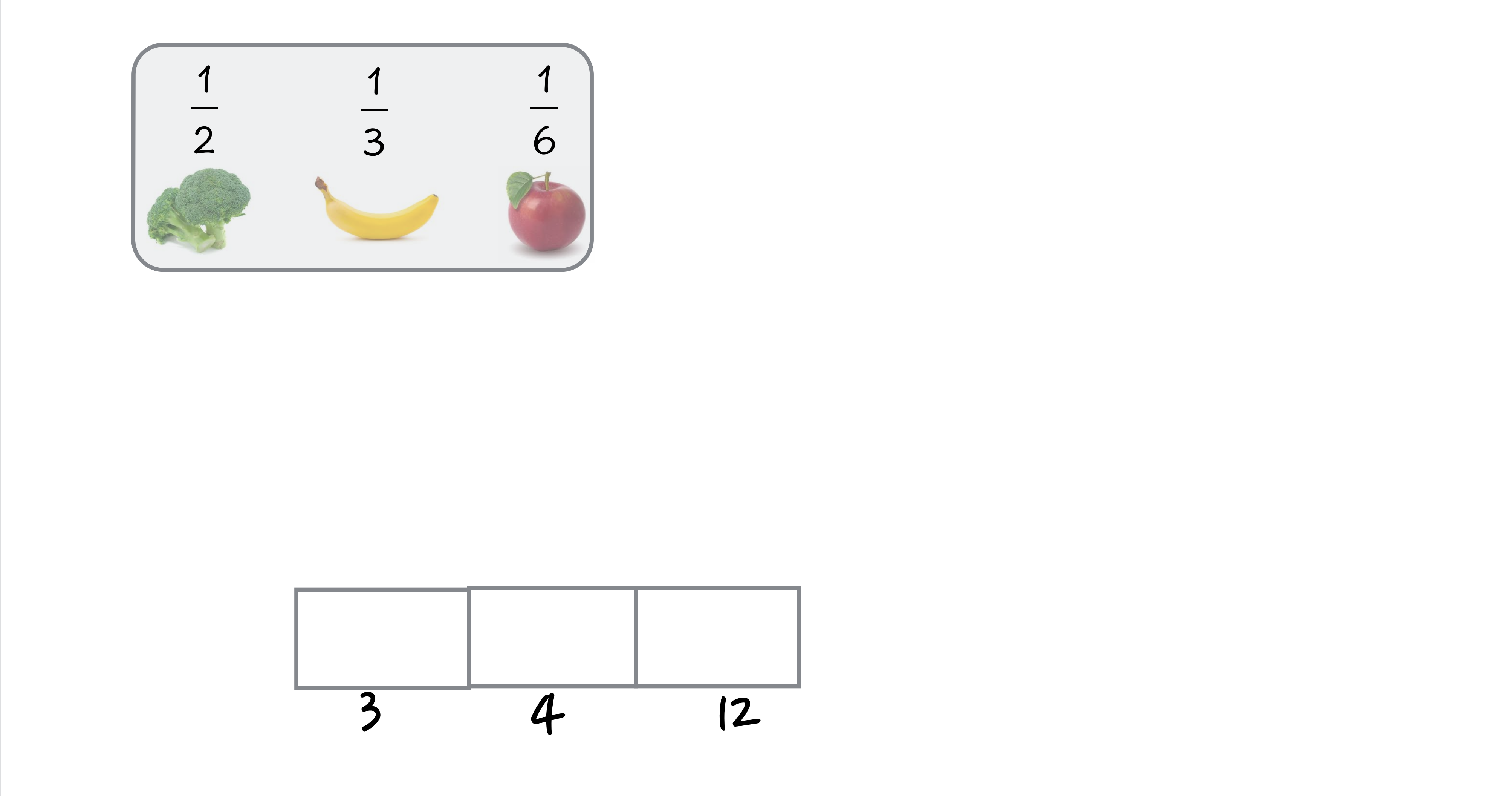
FcFS is just: sampling all items WITHOUT REPLACEMENT from p into slots.



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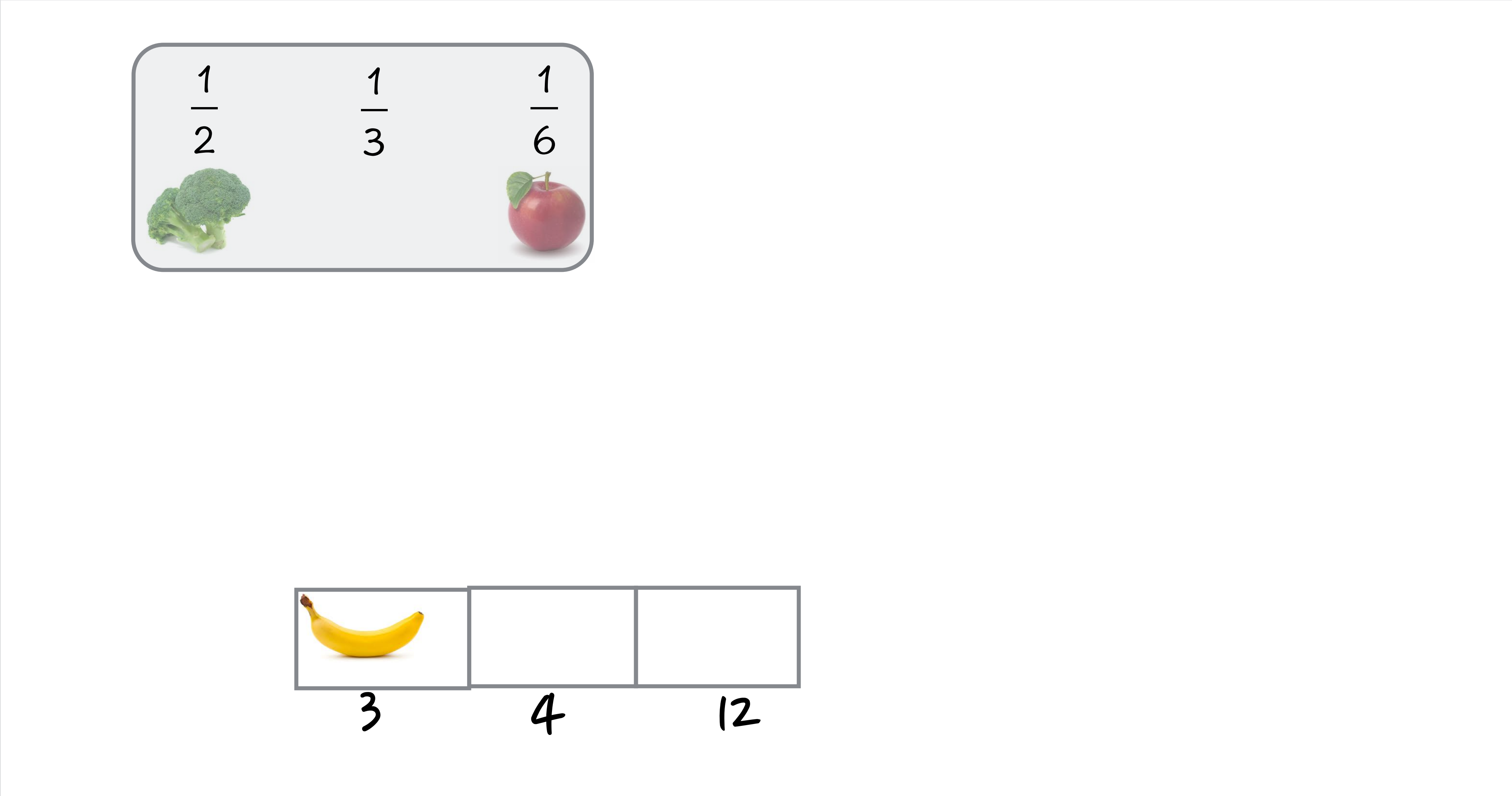

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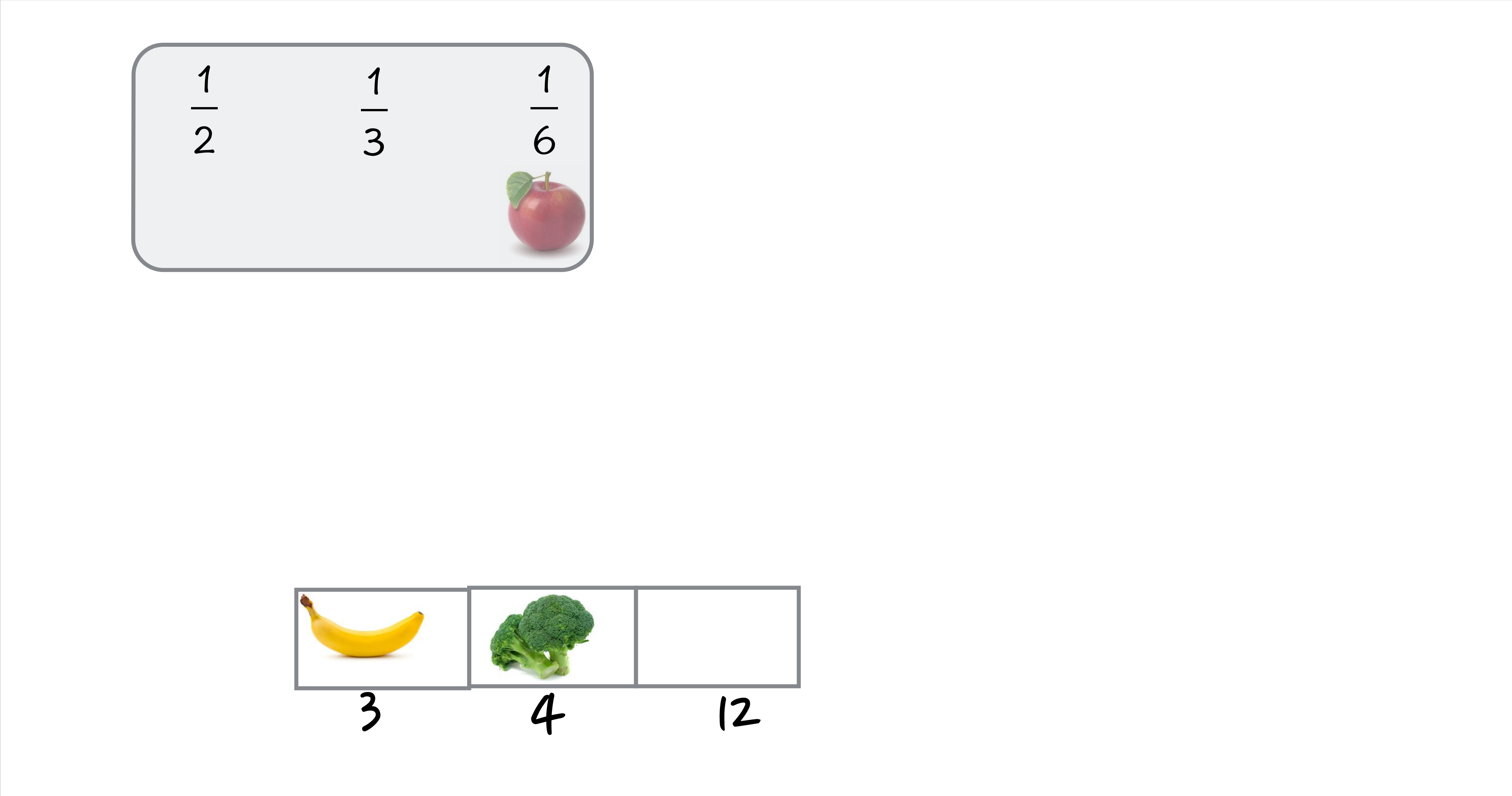

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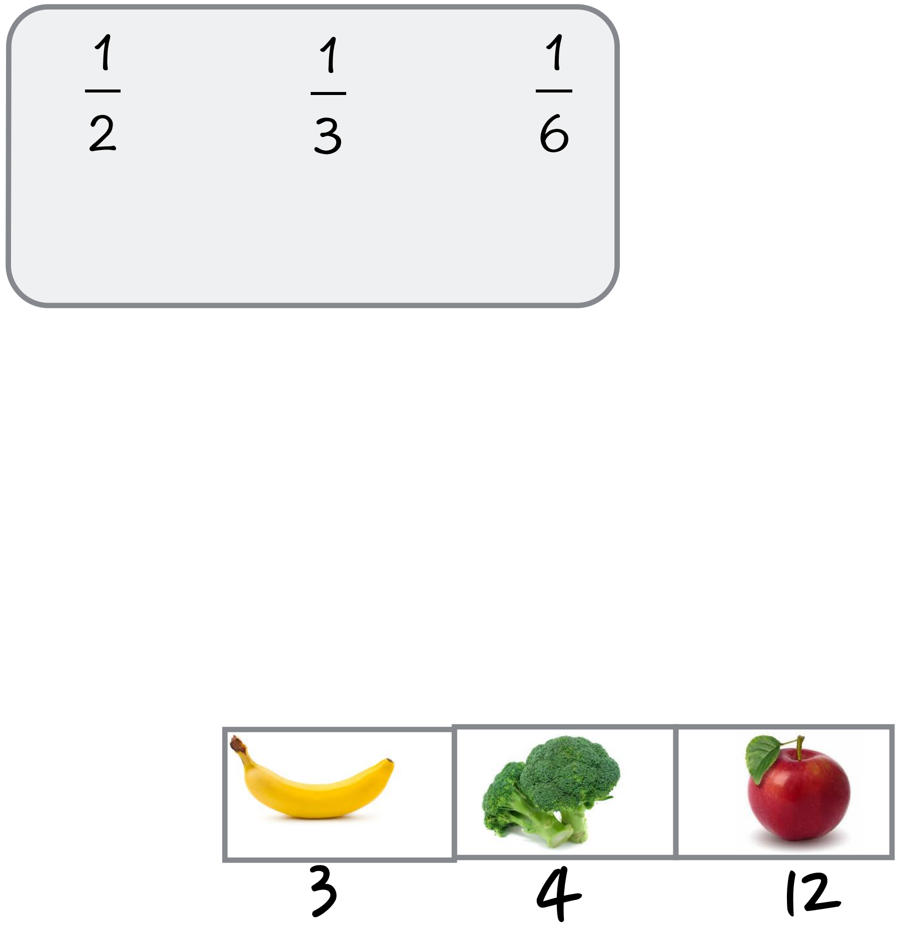

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main results

THM 1: FCFS is optimally competitive for online Slot Allocation.

TODAY

THM 2: optimal competitive ratios:

(a) Arbitrary slot costs: $1+H_{n-1}$

TODAY (most technically interesting)

(b) concave slot costs: 2

(c) Logarithmic slot costs: 1^*

*asymptotically: FCFS guarantees cost OPT + O(log OPT)..

THM 3: For online Huffman coding, online algorithm with cost

$$\mathrm{OPT} + 2 \log_2 (1 + \mathrm{OPT}) + 2.$$

(some) related work

competitive analysis w. STATIC OPT & unknown item distribution

- List management ~ OSA with linear cost function [34]
- Paging ~ OSA with 0/1 cost function
  (Independent Reference model — IRM) e.g. [1,12]

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ADAPTIVE Huffman coding [8,14,26,37,38,39]

- also one-pass, but codewords change adaptively.
- text can be arbitrarily ordered.

“WORST-DISTRIBUTION” COMPETITIVE ANALYSIS:

Oh no! Worst-case analysis is too pessimistic!

Oh no! Average-case analysis is too optimistic!

Show that your algorithm does well against any distribution in a class of distributions.

- competitive paging [23,28,33,42]
  (e.g. Markov paging, diffuse adversary)
- online bin packing [4,19]; online knapsack [30]
- online facility location, Steiner tree [32]
- Secretary problem [6,16]; online auctions [2,9,13]
- adwords [31,21,5]

sampling w/o replacement for poker tournaments

valuing chips = estimating, for sampling without replacement:

Pr[item i ends in slot j]

your expected final payout, given your current chips:

Pr[first place] * (payout for first place)

+ Pr[second place] * (payout for second place)

+ …

+ Pr[last place] * (payout for last place)

random model for your final placement, given current chips:

- round 1: select first-place player by random draw where

  Pr[player i wins first place] = players chips / total chips

- round 2: select second-place player from REMAINING players, again

  Pr[player i wins second place] = players chips / total chips of remaining players

- etc… = sampling players without replacement, using current chip counts as probabilities

  You finish in j'th place in tournament <==> You are the j'th sample

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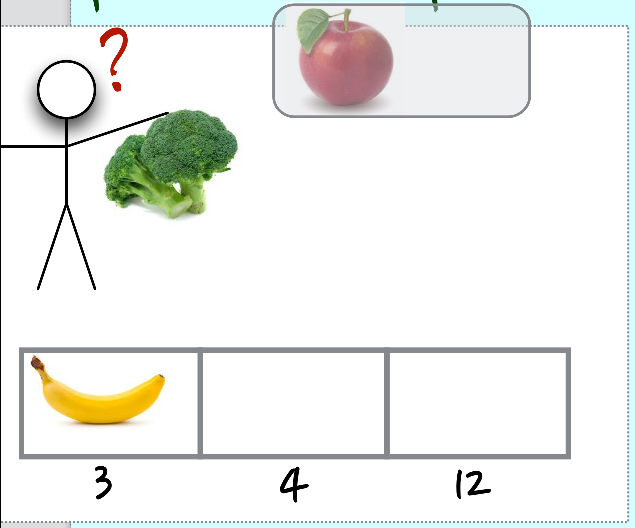
proof attempt 1



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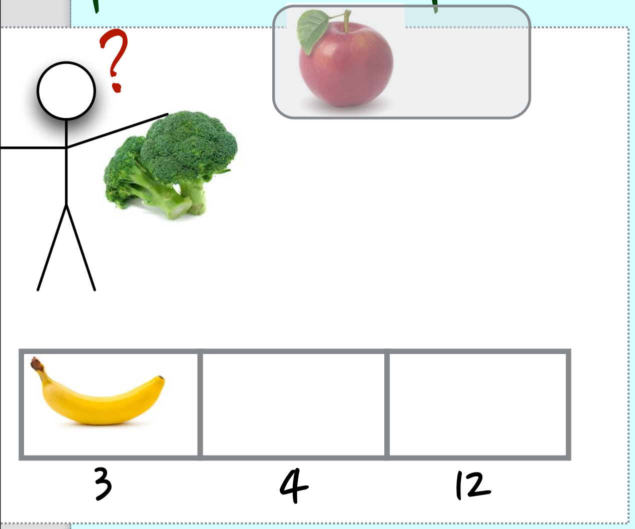

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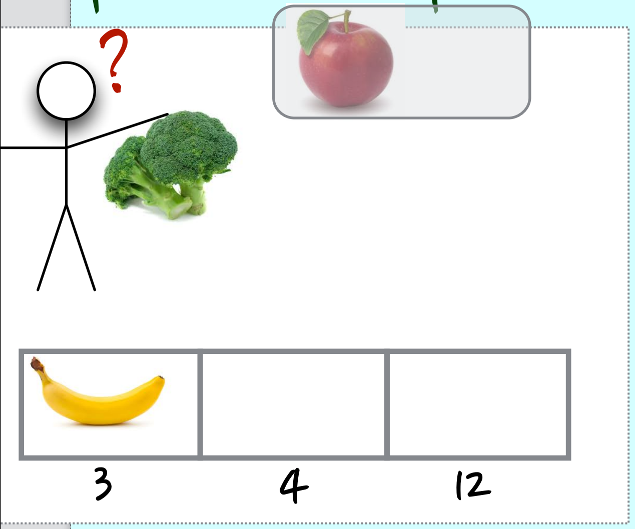

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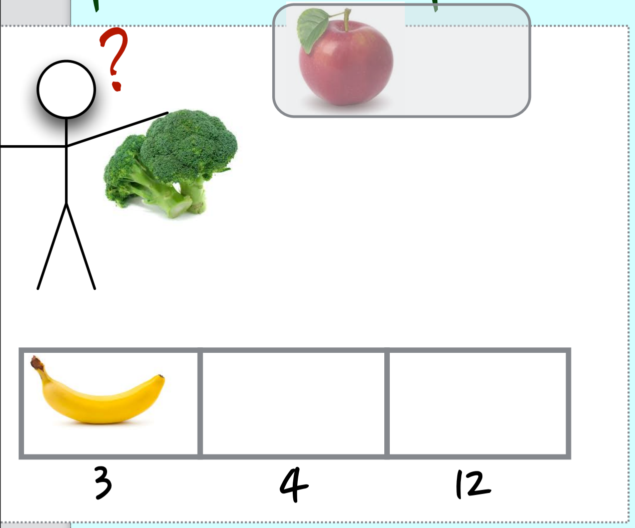

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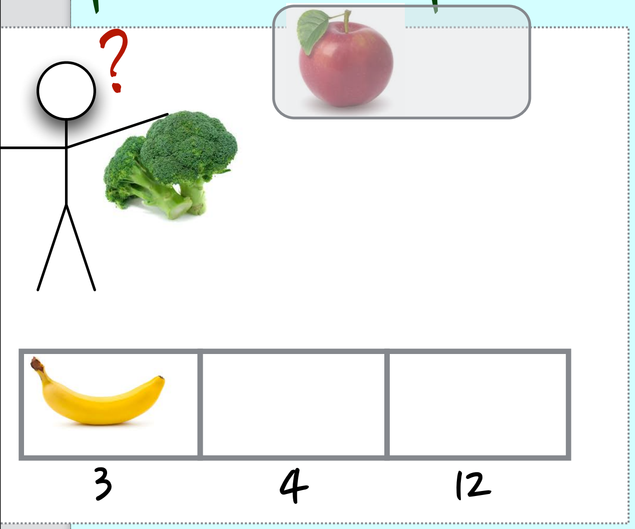

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FIX: Prove stronger result: FCFS best among WEAKLY ONLINE algorithms.

WEAKLY ONLINE = alg. KNOWS p but chooses next slot just BEFORE next request.

Now "more likely" is well-defined... (Rest of proof is technical but not surprising.)

THEOREM 2: Optimal competitive ratios for online Slot Allocation:

(a) arbitrary slot costs: $1 + H_{n-1}$

NEXT (UPPER BOUND ONLY)

(b) concave slot costs: 2

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*asymptotically: FCFS guarantees cost $\mathrm{OPT} + O(\log \mathrm{OPT})$.

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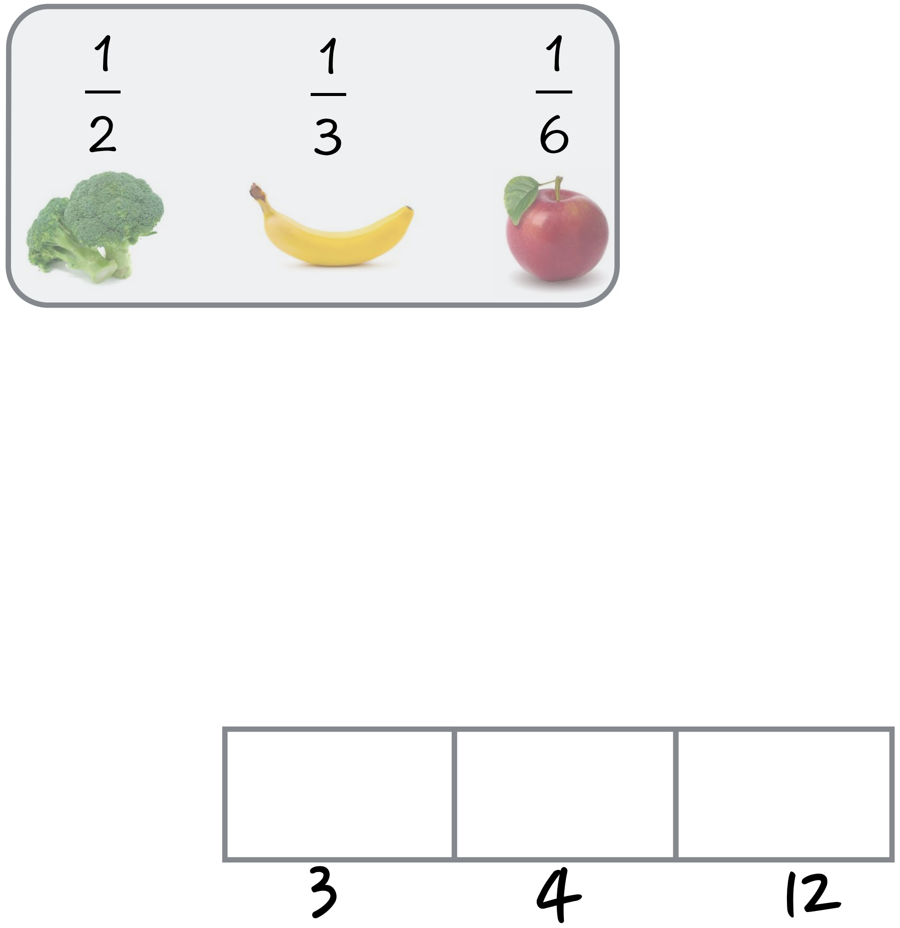
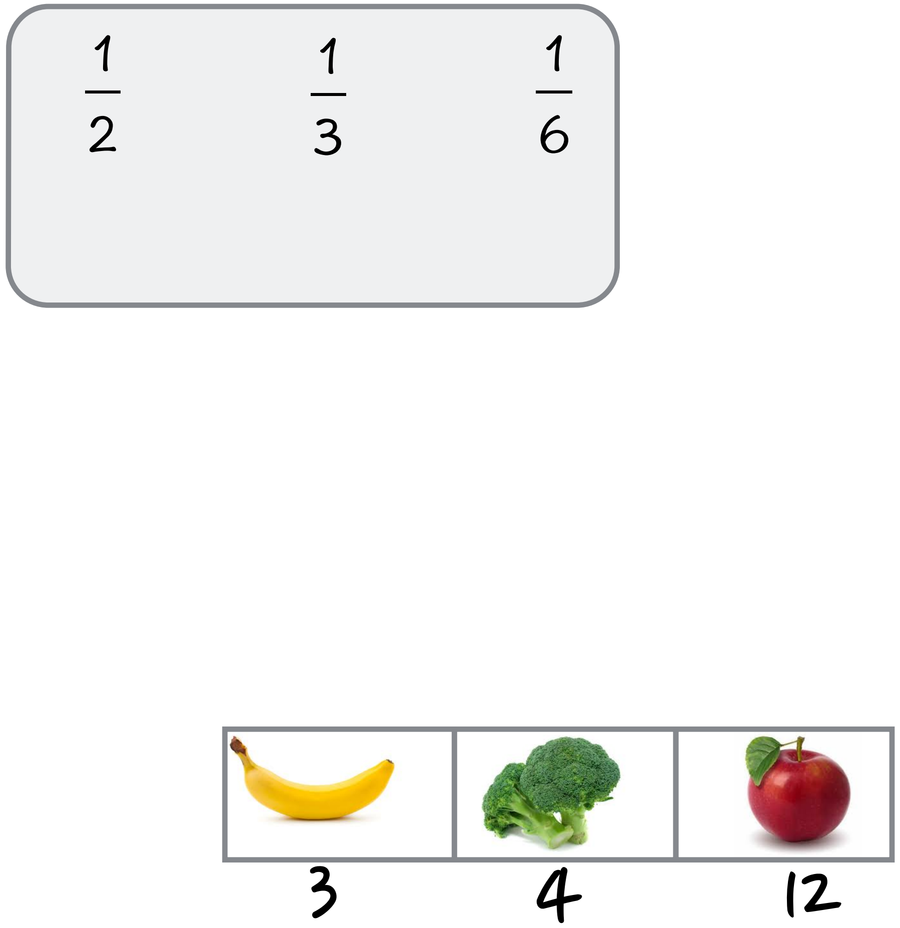


OPT is just: allocate highest-cost slots to lowest-probability items

FCFS is just: sampling all items WITHOUT REPLACEMENT from p into slots.



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THEOREM 2(a) upper bound: FCFS is $(1+H_{n-1})$-competitive. Proof idea

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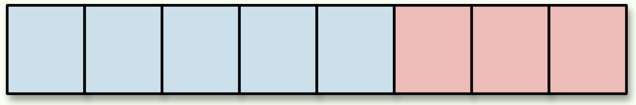

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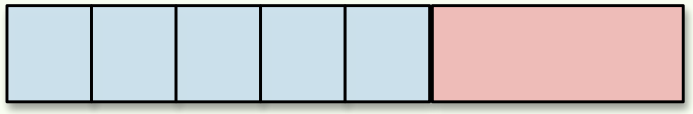

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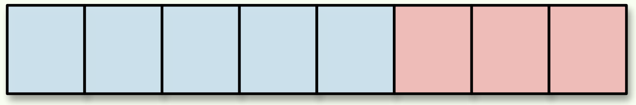
1. Example: five slots of cost 0, three slots of cost 1: 

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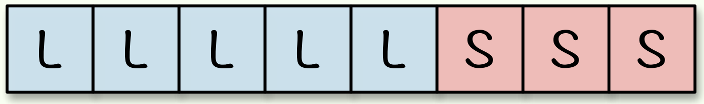
2. Optimal solution =  L L L L L S S S, OPT cost is S+S+S.

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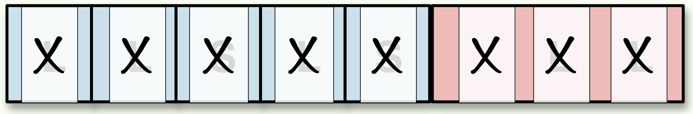
4. EXPOSURE view of FcFS:  X X X X X X X X

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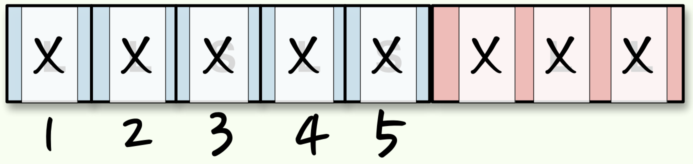

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1	2	3	4	5			

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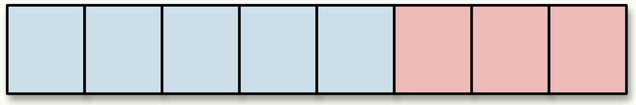
1. Example: five slots of cost 0, three slots of cost 1 : 

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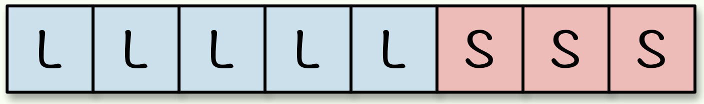
2. Optimal solution = 

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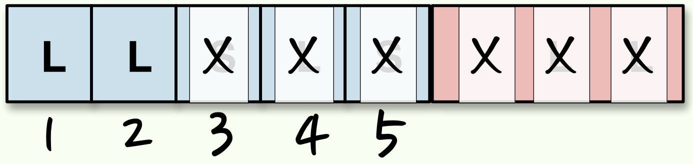
4. EXPOSURE view of FCFS: 

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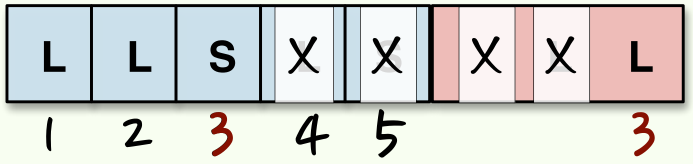

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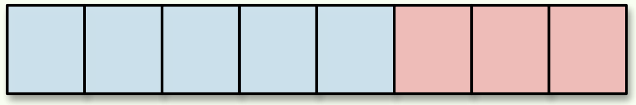

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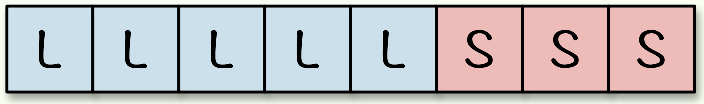

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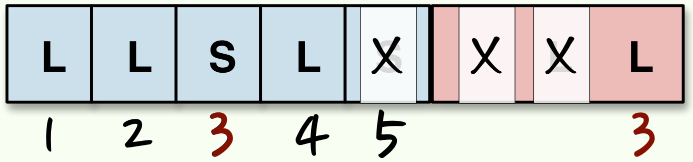

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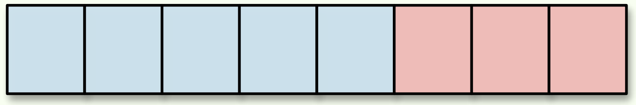

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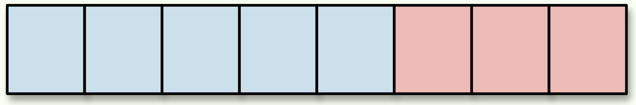

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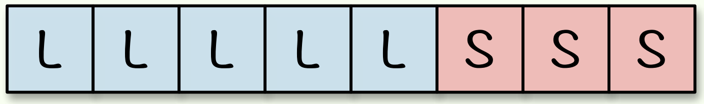

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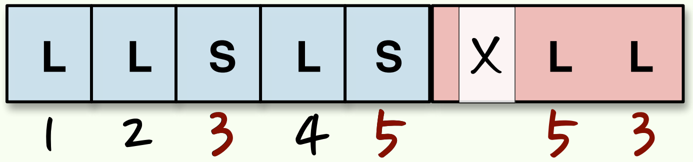

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proof idea
(continued)

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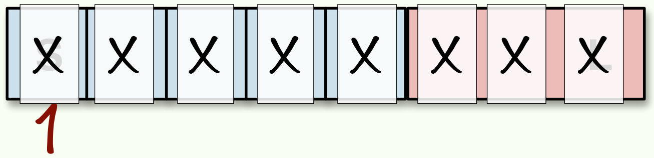

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(continued)

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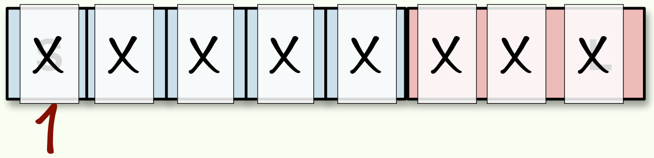

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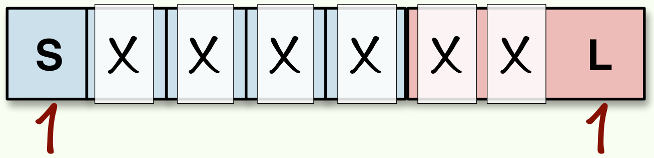

S X X X X X X L

1 1

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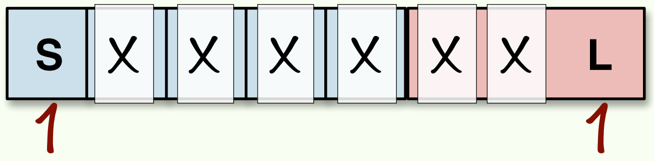

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$$\text{Pr[item 1 small]} \times E[\text{revealed large item cost}] \leq \frac{\text{OPT}}{\text{sum of all probabilities}} \times \frac{\text{sum of large probabilities}}{5}$$

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(continued)

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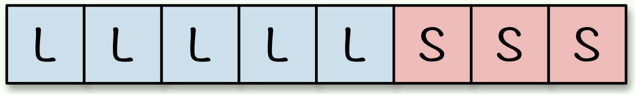

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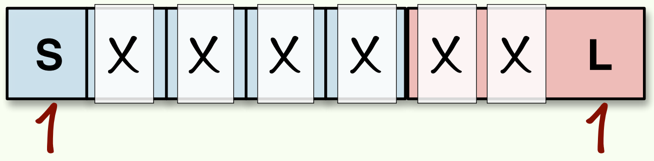

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$$\leq \frac{\text{OPT}}{5}$$

7. Second step: $\frac{\text{OPT}}{4}$; third: $\frac{\text{OPT}}{3}$; fourth: $\frac{\text{OPT}}{2}$; fifth $\frac{\text{OPT}}{1}$. Total $H_5 \times \text{OPT}$. QED ?

summary

THM 1: FcFS is optimally competitive for online Slot Allocation.

THM 2: optimal competitive ratios:

(a) Arbitrary slot costs: $1+H_{n-1}$

(b) concave slot costs: 2

(c) Logarithmic slot costs: 1^*

*asymptotically: FcFS guarantees cost OPT + O(log OPT)..

THM 3: For Huffman coding, online algorithm with cost

$$\mathrm{OPT} + 2\log_2(1 + \mathrm{OPT}) + 2.$$

sampling w/o replacement for poker tournaments

valuing chips = estimating, for sampling without replacement:

Pr[item i ends in slot j]

sampling w/o replacement for poker tournaments

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Pr[item i ends in slot j]

your expected final payout, given your current chips:

```
      Pr[   first place ] * (payout for first place)
+     Pr[second place ] * (payout for second place)
+     ...
+     Pr[    last place ] * (payout for last place)
```

sampling w/o replacement for poker tournaments

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your expected final payout, given your current chips:

Pr[first place] * (payout for first place)
+ Pr[second place] * (payout for second place)
+ …
+ Pr[last place] * (payout for last place)

random model for your final placement, given current chips:

- round 1: select first-place player by random draw where

  Pr[player i wins first place] = players chips / total chips

- round 2: select second-place player from REMAINING players, again

  Pr[player i wins second place] = players chips / total chips of remaining players

- etc… = sampling players without replacement, using current chip counts as probabilities

You finish in j'th place in tournament <==> You are the j'th sample

GIVEN: slots with costs $c(1) \le c(2) \le \cdots \le c(n)$, requests i

ALLOCATE: slot j

OBJECTIVE: Minimize cost $\sum_{i=1}^{n} p_i \, c(j_i)$

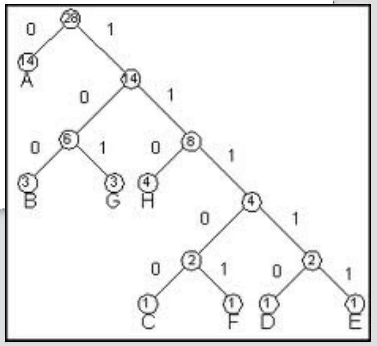
defn of OSA

DEFN OF ONLINE HUFFMAN CODING

GIVEN: letters i_1, i_2, i_3, $\cdots$ i.i.d. from unknown distribution p.

ALLOCATE: codeword j_i for each letter i on first occurrence.

OBJECTIVE: Minimize cost $\sum_{i=1}^{n} p_i \, c(j_i)$ $\quad (c(j) \approx \log_2 j)$



First-Come First-Served (FCFS) for Online Slot Allocation (OSA) and Huffman Coding

— SODA 2014 —

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