Computing Simple Paths Among Obstacles

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Abstract

Given a set X of points in the plane, two distinguished points $s, t \in X$, and a set Φ of obstacles represented by line segments, we wish to compute a simple polygonal path from s to t that uses only points in X as vertices and avoids the obstacles in Φ. We present two results: (1) We show that finding such simple paths among arbitrary obstacles is NP-complete. (2) We give a polynomial-time algorithm that computes simple paths when the obstacles form a simple polygon P and X is inside P. Our algorithm runs in time $O(m^2n^2)$, where m is the number of vertices of P and n is the number of points in X.

1 Introduction

The research we describe in this paper was motivated by polygon generation problems. Suppose that given a set X of points in the plane, we wish to generate all simple polygons whose vertices are in X. A simple, iterative approach to this problem is to start at an arbitrary point $x \in X$ and successively extend the path. Given the current path L ending at a point y, let Y denote the set of points of X that are not on L. We can extend L to all points $z \in Y \cup \{x\}$ for which the interior of the line segment $[y, z]$ does not intersect L. For some sequences of choices, however, we can reach situations when y cannot be connected to any $z \in Y \cup \{x\}$ without intersecting L. Thus such a path L is not a feasible partial solution. In order to avoid this difficulty, and reduce the backtracking, at any step of this process, given the current (feasible) path L, we would like to verify which extensions of L are feasible, that is, to compute the set of all z's in Y for which the extended path $L' = Lz$ can be completed to a simple polygon. One can think of this question as the problem of finding a simple path that avoids given obstacles and uses given points as vertices: we want to find a simple path from z to x that does not intersect $L' - \{x, z\}$ and whose vertices are in $Y \cup \{x\}$.

A similar idea can be applied to the problem of generating a "random" polygon with vertices

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