A Simple Analysis of the Harmonic Algorithm for Two Servers

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1 Introduction

In the *k-server problem* we are given a metric space M populated by k mobile serves. At each time step a request $r \in M$ is specified, and we need to move one server to r to "satisfy" the request. The decision as to which server to move needs to be made *online*, without the knowledge about future requests.

An online server algorithm A for the k-server problem is called *C-competitive* if its cost on any request sequence ϱ is at most C times the optimal (offline) cost on this sequence, plus possibly an additive constant independent of ϱ. The same definition applies to randomized algorithms as well, we just need to replace our cost by our *expected* cost on ϱ. The *competitive ratio* of A is the smallest C for which A is C-competitive.

Manasse, McGeoch and Sleator [6] gave a 2-competitive algorithm for 2 servers and they proved that no deterministic online algorithm for k servers can be better than k-competitive. They also formulated the still-open *k-server conjecture* that a k-competitive algorithm exists for each k. The best known upper

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