CS133 Assignment 2

Due date: Thursday 4/25/2019 at 11:59 PM

Convex Hull

1. (2 points) Given a list of points, develop a linear time algorithm that tests whether the points form a convex hull or not. Notice that the points might come in either CW or CCW order. In both cases, the algorithm should return true as long as they form a convex hull.
   Example 1: Input: [(0,0), (1,0), (2,2)] ➔ Output: True
   Example 2: Input: [(0,0), (2,0), (1,1), (1,3)] ➔ Output: False
   Example 3: Input: [(0,0), (1,1), (2,2), (1,0)] ➔ Output: True
2. (3 points) Given a set of points P and a straight line defined by two points p_1 and p_2, prove that the point $p_i \in P$ that is farthest away from the line $\overline{p_1p_2}$ is part of the convex hull of P. This can also be expressed using the following mathematical expression.
   $$p_i \in P \wedge p_j \in P \wedge dist(p_i, \overline{p_1p_2}) \geq dist(p_j, \overline{p_1p_2}) \Rightarrow p_i \in \mathcal{CH}(P)$$
   where $dist(p, \bar{l})$ is the Euclidean distance between the point p and its projection $p`$ on the line $\bar{l}$ and $\mathcal{CH}(P)$ is the convex hull of P.
   This proof is needed for the recursive part of the Quick Hull algorithm.
3. (2 points) Building on your proof in 2, prove that the farthest pair of points p_1 and p_2 in a set P have to be both on the convex hull of P. This can be expressed using the following expression.
   $$p_{i,j,k,l} \in P \wedge dist(p_i, p_j) \geq dist(p_k, p_l) \Rightarrow p_{i,j} \in CH(P)$$
   where $p_{i,j,k,l} \in P$ means that all of the points p_i, p_j, p_k, and p_l are in the set P
4. (3 points) Describe how to craft a worst-case input of size n points for the Quick Hull algorithm. Recall that a worst-case scenario of the Quick Hull algorithm yields an $O(n^2)$ asymptotic running time.