AVL Trees

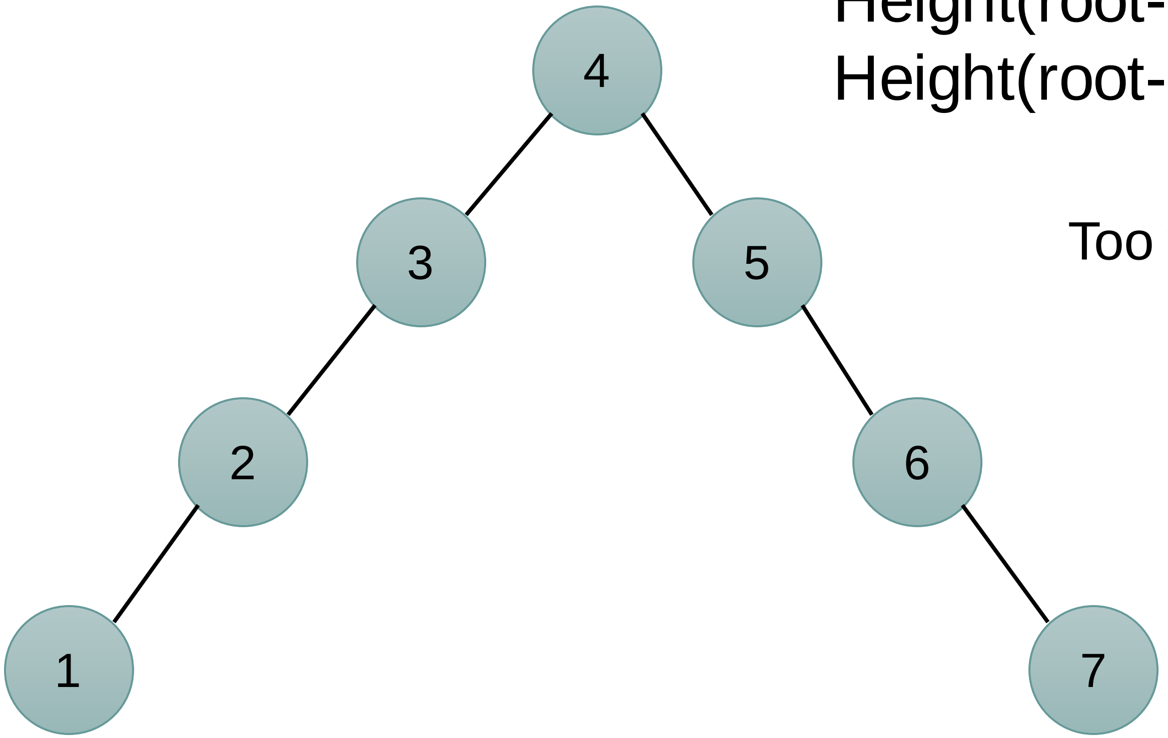
Section 4.4

AVL Tree

- A balanced tree
- Ensures $O(\log n)$ running time for search, insert, and delete
- A simple and relaxed definition for balance
- $\lfloor \log n \rfloor \le h \le \lceil \log n \rceil$: Too restrictive

Balanced Tree

Height(null) = -1

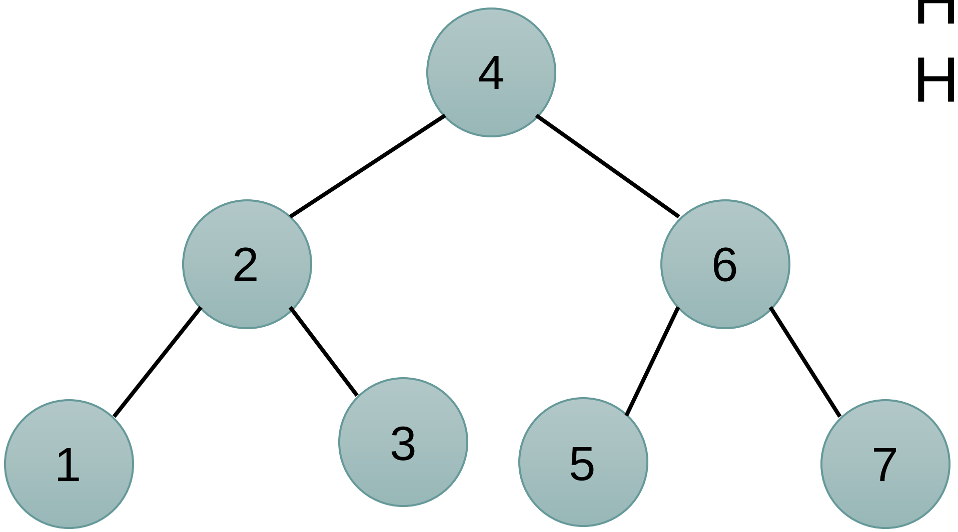


Height(root->left) =
Height(root->right)

Too weak

Balanced Tree

Height(null) = -1

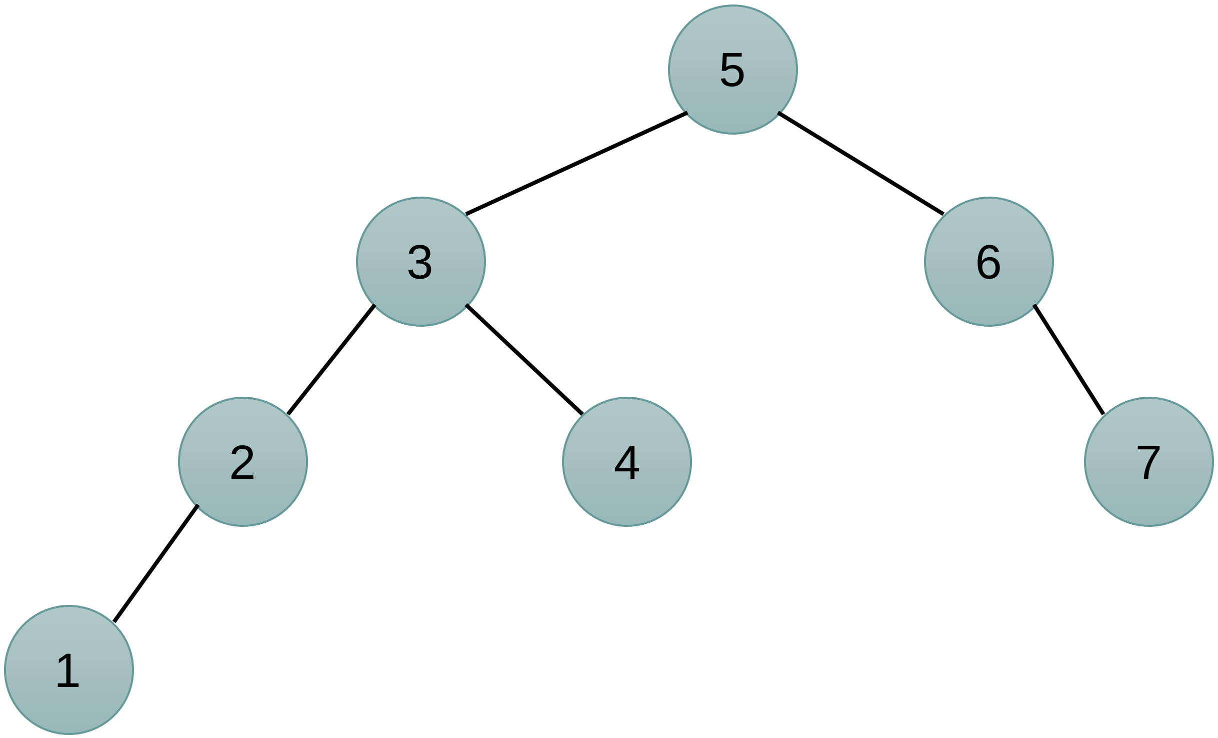


Height(node->left) = Height(node->right)

Could be impossible to satisfy

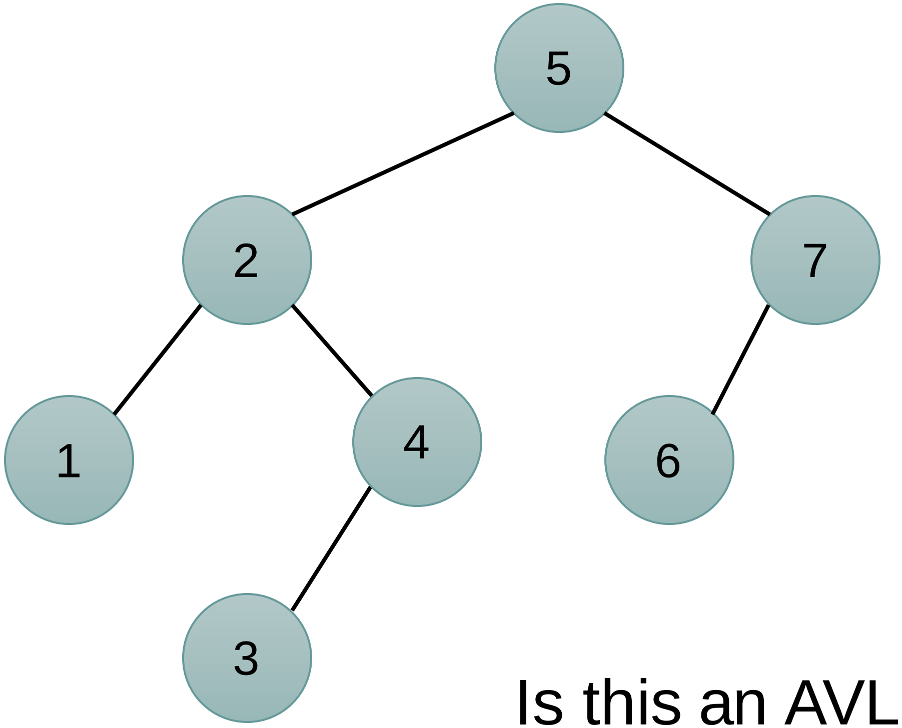
AVL Balance Condition

$$|Height(node \rightarrow left) - Height(node \rightarrow right)| \leq 1$$



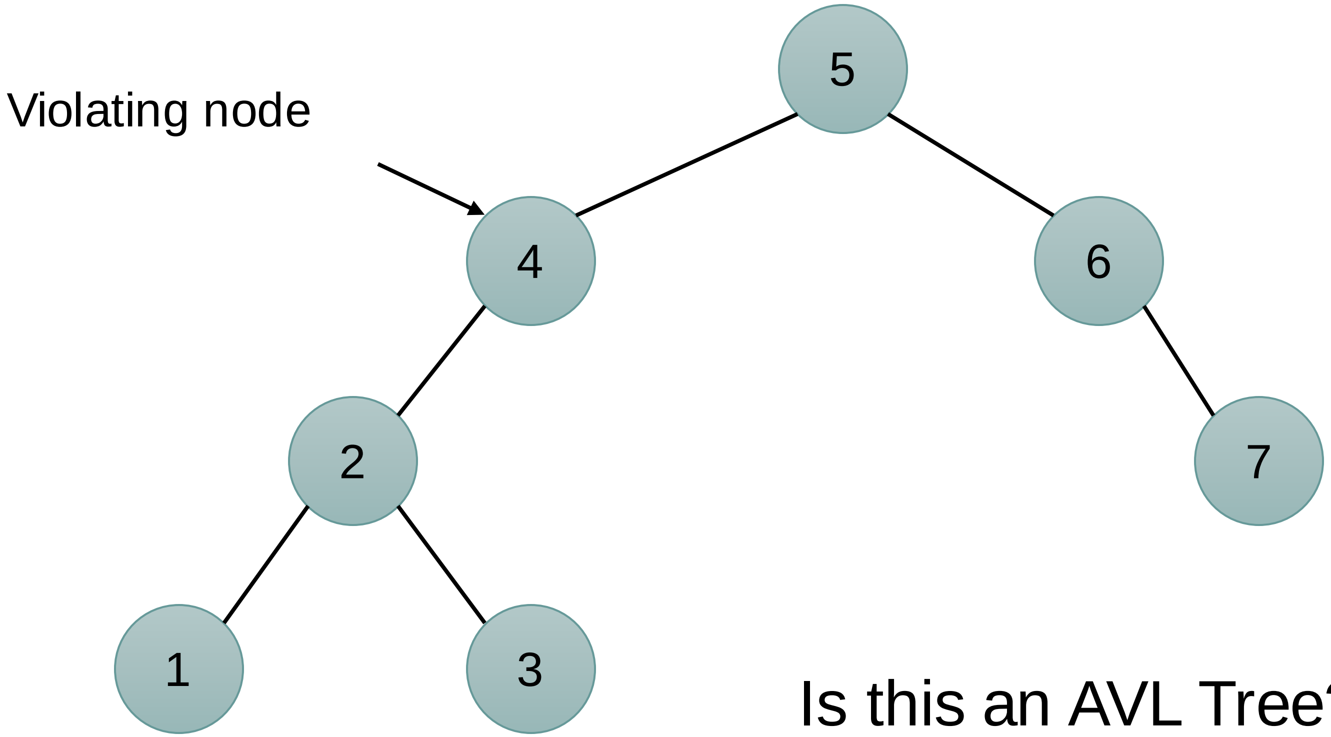
Height(null) = -1

AVL Example



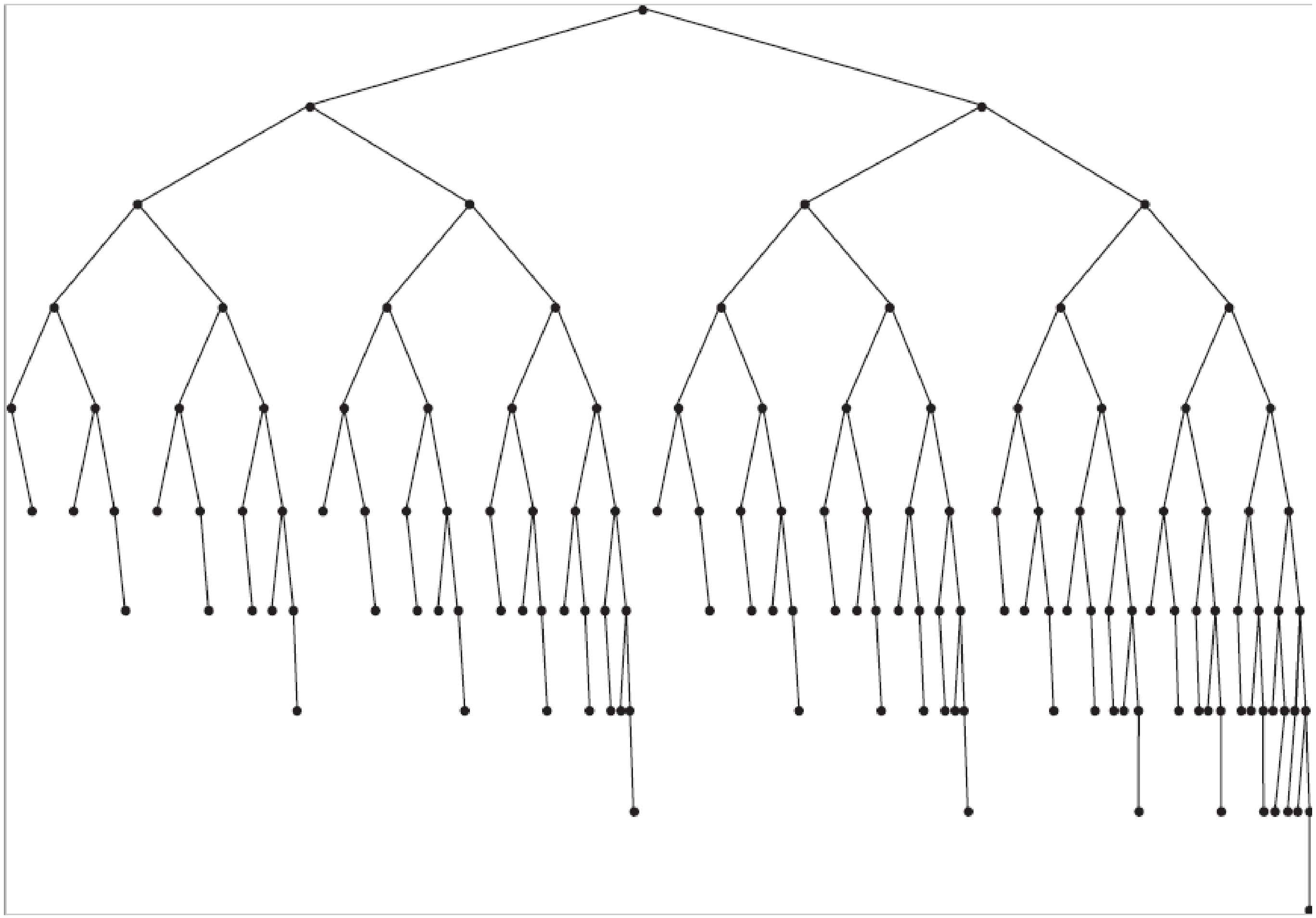
Is this an AVL Tree? Yes

AVL Example



Is this an AVL Tree? No

AVL Example



Is this an AVL Tree? Yes

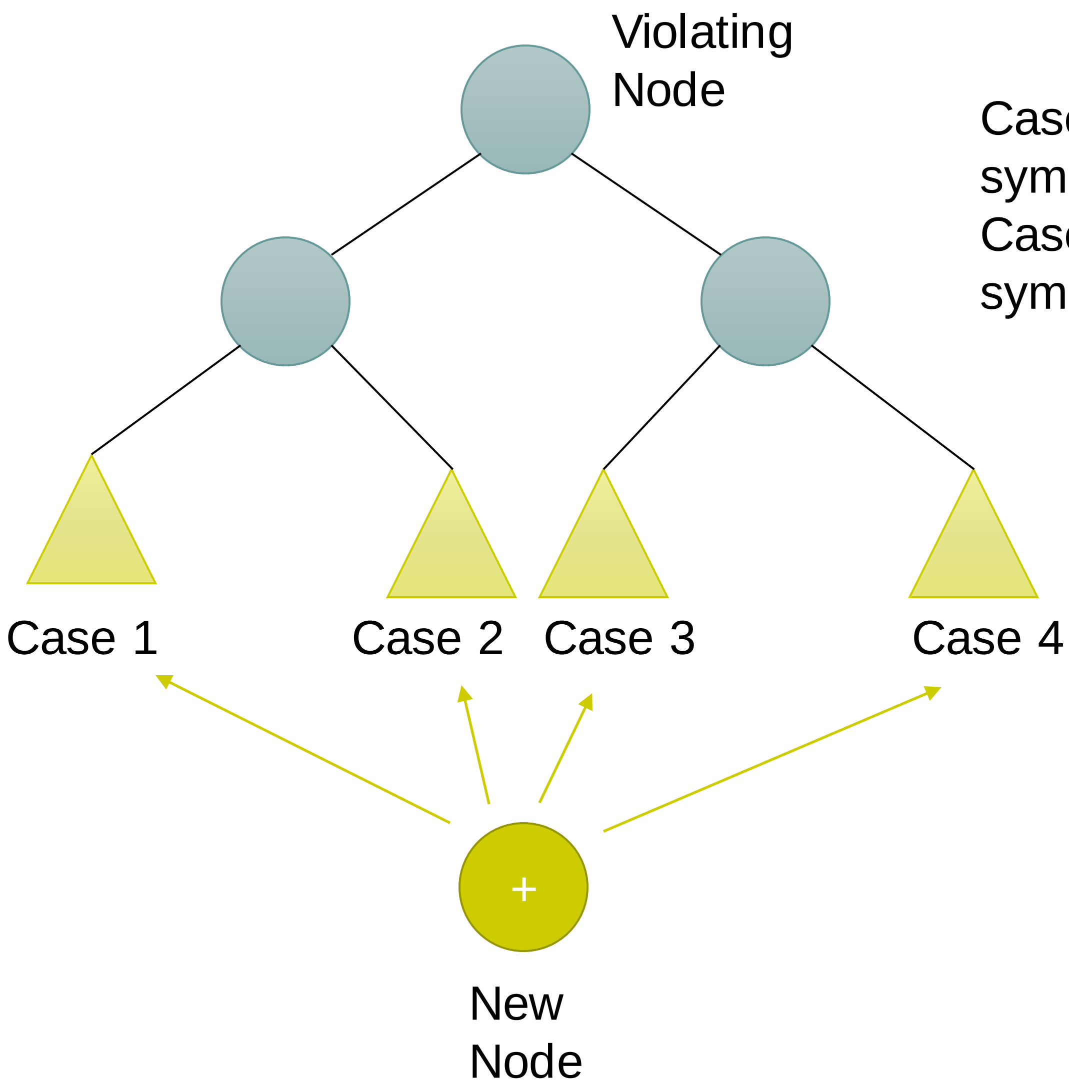
Balancing an AVL Tree

- For simplicity, we assume that we keep the height of each subtree at its root
- An imbalance can occur as a result of an insertion or deletion
- To balance an AVL tree, we carry out a **rotation operation**

Insertion

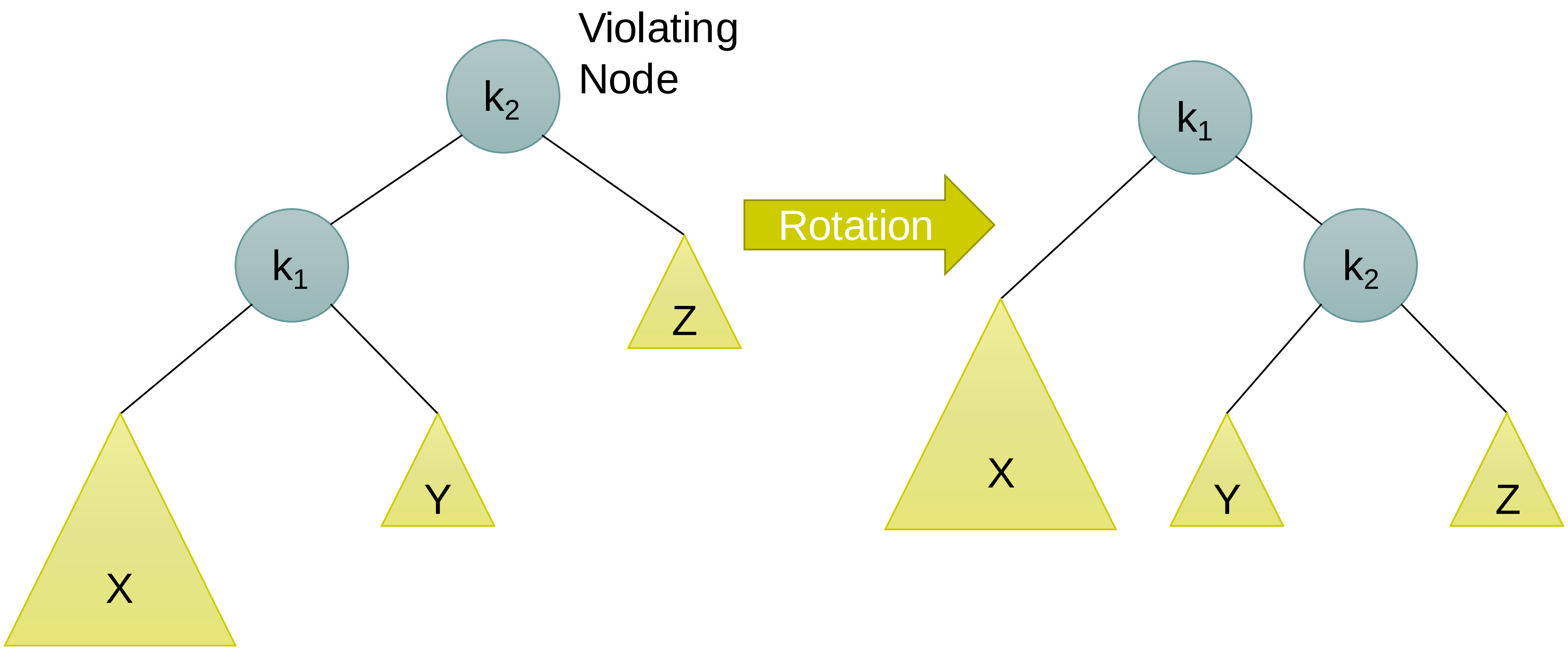
- Call BST.insert
- Update the height as you climb up to the root
- After each height update, check for an AVL tree violation and fix using rotation

Violation after Insertion



Cases 1 and 4 are symmetric
Cases 2 and 3 are symmetric

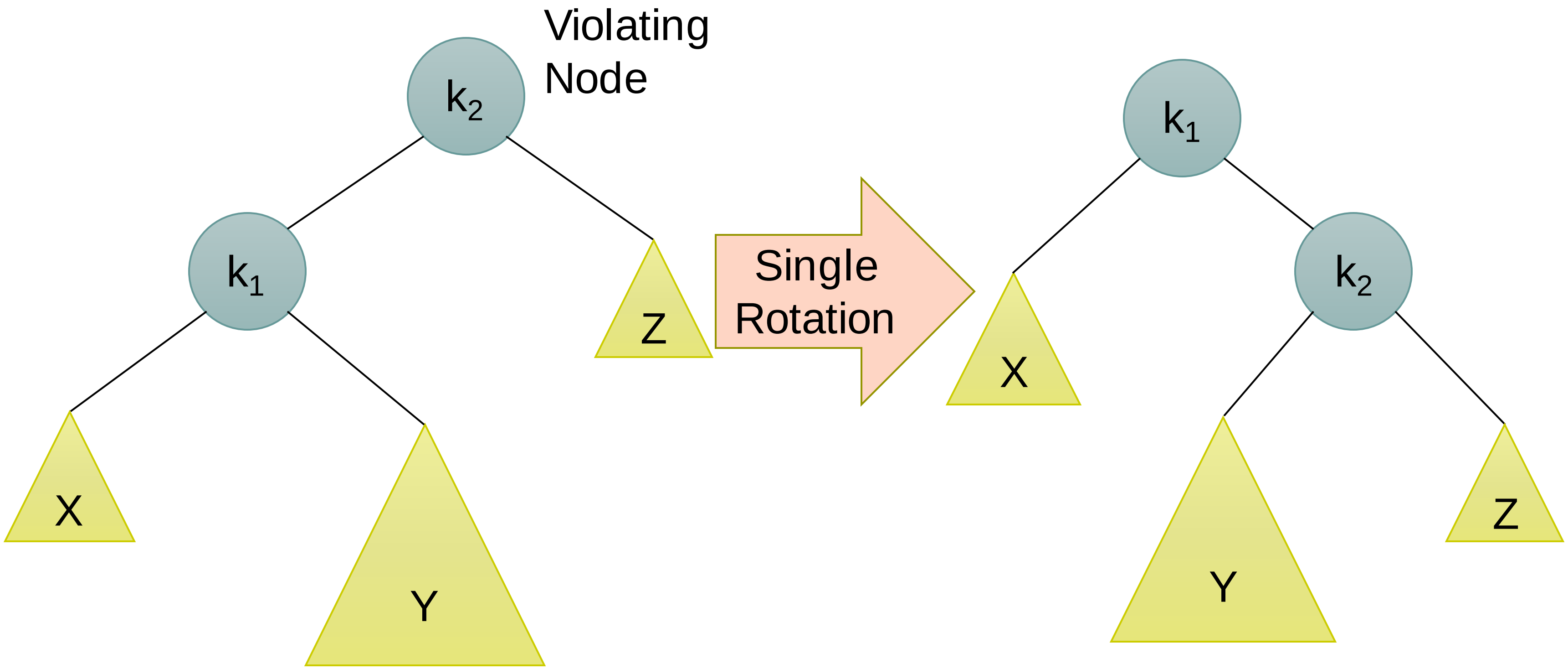
Case 1 – Single Rotation



Status upon insertion in X
k_2 is in violation

Is this a BST? Yes
Is this an AVL Tree? Yes

Case 2 – Single Rotation?



Status upon insertion in Y
k_2 is in violation

Is this a BST? Yes
Is this an AVL Tree? No

Case 2 – Double Rotation